Achieve ADP Algebra II
End-of-Course Exam Content Standards
with Comments & Examples

Core and Optional Modules

January 2010

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About this document:

This version of the Algebra II End-of-Course Exam Content Standards includes two columns: The first column contains the standards, objectives and benchmarks; the second column includes explanatory comments and examples. These comments and examples are meant to add clarity to the meaning of the benchmarks for teachers and test item writers. Examples are provided only when necessary for clarity and are not meant to be exhaustive or to be used as sample test items. In some instances, in the standards and in the explanations, the word "including" is used followed by a list. The word "including" does not translate to "all inclusive" but rather means "including but not limited to." Some of the benchmarks have "assessment limitations" which means that the stated content is not tested on the ADP Algebra II End-of-Course Exam. However, this does not imply that teachers should not teach or expand on this content.

Background: The American Diploma Project (ADP) Network includes states dedicated to making sure every high school graduate is prepared for college and a career. In each state, governors, state superintendents of education, business executives, and college and university leaders are working to restore value to the high school diploma by raising the rigor of high school standards, assessments, and curriculum, and better aligning these expectations with the demands of postsecondary education and careers.

In May 2005, leaders from several of the ADP Network states began to explore the possibility of working together, with support from Achieve, to develop a common end-of-course exam at the Algebra II level. These states were planning to require or strongly encourage students to take an Algebra II level course in order to better prepare them for college and careers, as Algebra II or its equivalent serves as a gateway course for higher education and teaches quantitative reasoning skills important for the workplace. State leaders recognized that using a common end-of-course test would help ensure a consistent level of content and rigor in classes within and across their respective states. They also understood the value of working collaboratively on a common test: the potential to create a high quality test faster and at lower cost to each state, and to compare their performance and progress with one another.

The ADP Algebra II End-of-Course Exam serves three main purposes:

1. ***To improve curriculum and instruction**—and ensure consistency within and across states***.** The test will help classroom teachers focus on the most important concepts and skills in an Algebra II or equivalent class and identify areas where the curriculum needs to be strengthened. Teachers will get test results back within three weeks of administering the test, which will provide sufficient time to make the necessary adjustments for the next year's course.

2. ***To help colleges determine if students are ready to do credit-bearing work.*** Because the test is aligned with the ADP mathematics benchmarks, it will measure skills students need to enter and succeed in first-year, credit-bearing math courses. Postsecondary institutions will be able to use the results of the test to tell high school students whether they are ready for college-level work, or if they have content and skill gaps that need to be filled before they enroll in college. This information should help high schools better prepare their students for college, and reduce the need for colleges to provide costly remediation courses.

3. ***To compare performance and progress among the participating states.*** Having agreed on the core content expectations for courses at the Algebra II level, states are interested in tracking student performance over time. Achieve will issue a report each year comparing performance and progress among the participating states. This report will help state education leaders, educators and the public assess performance, identify areas for improvement, and evaluate the impact of state strategies for improving secondary math achievement.

The Algebra II End-of-Course Exam will consist of a common core, which will be taken by students across participating states. States not part of the development group may also decide to purchase and administer this test. In addition to the core algebraic content, seven optional modules will be available to states to enrich the core with content which may currently be in their Algebra II or equivalent course that is deemed important to colleges and the workplace. The content in the core and modules is described below.

The Core Algebra II End-of-Course Exam: The core Algebra II End-of-Course Exam covers a range of algebraic topics. Successful students will demonstrate conceptual understanding of the properties and operations of real and complex numbers. They will be able to make generalizations through the use of variables resulting in facility with algebraic expressions. They will solve single and systems of linear equations and inequalities and will be able to use them to represent contextual situations. Successful students also will be able to demonstrate facility with estimating and verifying solutions of various non-linear equations, making use of technology where appropriate to do so. Finally, students will demonstrate knowledge of functions and their properties – distinguishing among quadratic, higher-order polynomial, exponential, and piecewise-defined functions – and recognize and solve problems that can be modeled by these functions. They will be required to analyze these models, both symbolically and graphically, and to determine and effectively represent their solution(s). There are a variety of types of test items developed that will assess this content, including some that cut across the objectives in a standard and require students to make connections and, where appropriate, solve rich contextual problems.

There are three types of items on the Algebra II End-of-Course Exam, including multiple choice (worth 1 point each), short answer (worth 2 points each), and extended response (worth 4 points each). At least one-third of the student's score will be based on the combined scores of the short-answer and extended-response items. Although the test is untimed, it is designed to take approximately 180 minutes, comprised of two 90 minute sessions, one of which will allow calculator use. Test items, in particular extended-response items, may address more than one content objective and benchmark within a standard. Each standard within the exam is assigned a priority, indicating the approximate percentage of points allocated to that standard on the test.

Optional Algebra II End-of-Course Exam modules: Optional modules in Data and Statistics, Probability, Logarithmic Functions, Trigonometric Functions, Matrices, Conic Sections, and Sequences and Series have been developed. One or more of these modules may be used by states, districts, or schools whose curriculum includes these topic areas in Algebra II or an equivalent course. Each module will include the same types of items as the core exam: multiple-choice, short answer, and extended response. Fifty percent of the student's score on the modules will be based on the combined scores of the short answer and extended response items. A student should be allowed at least 30 minutes for each module. Calculators are allowed on all of the modules.

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Algebra II End-of-Course Exam calculator policy: The appropriate and effective use of technology is an essential practice in the Algebra II classroom. At the same time students should learn to work mathematically without the use of technology. Computing mentally or with paper and pencil is required on the Algebra II End-of-Course Exam and should be expected in classrooms where students are working at the Algebra II level. It is therefore important that the Algebra II End-of-Course Exam reflects both practices. For purposes of the Algebra II End-of-Course Exam, students are expected to have access to a calculator for one of the two testing sessions and the use of a graphing calculator is strongly recommended. Scientific or four-function calculators are permitted but not recommended because they do not have graphing capabilities. Students should not use a calculator that is new or different for them on the Exam but rather should use the calculator they are accustomed to and use every day in their classroom work. For more information about technology use on the Algebra II End-of-Course Exam, see the ADP Algebra End-of-Course Exams Calculator Policy at www.achieve.org/AssessmentCalcPolicy.

It will be necessary to clear the calculator memory, including any stored programs and applications, on all calculators both before and after the exam. Please be advised that the clearing of the calculator memory may permanently delete stored programs or applications. Students should be told prior to the test day to store all data and software they wish to save on a computer or a calculator not being used for the test. In some states, an IEP or 504 Plan may specify a student's calculator use on this Exam. Please check with your state's Department of Education for specific policies or laws.

Algebra II level curriculum: Function modeling and problem solving is the heart of the curriculum at the Algebra II level. Mathematical modeling consists of recognizing and clarifying mathematical structures that are embedded in other contexts, formulating a problem in mathematical terms, using mathematical strategies to reach a solution, and interpreting the solution in the context of the original problem. Students must be able to solve practical problems, representing and analyzing the situation using symbols, graphs, tables, or diagrams. They must effectively distinguish relevant from irrelevant information, identify missing information, acquire needed information, and decide whether an exact or approximate answer is appropriate, with attention paid to the appropriate level of precision. After solving a problem and interpreting the solution in terms of the context of the problem, they must check the reasonableness of the results and devise independent ways of verifying the results.

The standards included in this document are intended to reflect this curricular focus and to guide the work of the test designers and the test item developers. It is also the case that curriculum at the Algebra II level will include content and processes not included on the Algebra II End-of-Course Exam, as some are not easily assessed on an end-of-course exam of this nature. Problems that require extended time for solution should also be addressed in the Algebra II level classroom, even though they cannot be included in this end-of-course exam.

Algebra II level classroom practices: Effective communication using the language of mathematics is essential in a class engaged in Algebra II level content. Correct use of mathematical definitions, notation, terminology, syntax, and logic should be required in all work at the Algebra II level. Students should be able to translate among and use multiple representations of functions fluidly and fluently. They should be able to report and justify their work and results effectively. To the degree possible, these elements of effective classroom practice are reflected in these Algebra II End-of-Course Exam content standards.

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Core: Operations on Numbers and Expressions

Priority: 15%

Successful students will be able to perform operations with rational, real, and complex numbers, using both numeric and algebraic expressions, including expressions involving exponents and roots. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems.

Content Benchmarks	Explanatory Comments and Examples
O1. Real numbers	
a. Convert between and among radical and exponential forms of numerical expressions.	• Convert between expressions involving rational exponents and those involving roots and integral powers. *Example*: $5^{\frac{3}{2}} = \sqrt{5^3} = 5\sqrt{5}$; $\sqrt[4]{27} = \sqrt[4]{3^3} = 3^{\frac{3}{4}}$.
b. Simplify and perform operations on numerical expressions containing radicals.	• Convert radicals to alternate forms and use the understanding of this conversion to perform calculations with numbers in radical form. *Example*: $\sqrt{8} + \sqrt{18} = 2\sqrt{2} + 3\sqrt{2} = 5\sqrt{2}$ • Divide irrational numbers, involving radicals, using conjugates, such as $a + \sqrt{b}$ and $a - \sqrt{b}$. *Example*: $\sqrt[3]{1{,}024} + \sqrt[3]{16} + \sqrt[5]{4{,}096} = 8\sqrt[3]{2} + 2\sqrt[3]{2} + 4\sqrt[5]{4} = 10\sqrt[3]{2} + 4\sqrt[5]{4}$
c. Apply the laws of exponents to numerical expressions with rational and negative exponents to order and rewrite them in alternative forms.	• Apply the properties of exponents in numerical expressions. *Example*: $3^5 \cdot 3^2 = 3^{(5+2)} = 3^7 = 2187$, $\frac{3^5}{3^2} = 3^{(5-2)} = 3^3 = 27$, $(3^5)^2 = 3^{(5\bullet 2)} = 3^{10} = 59049$, $3^{\frac{5}{2}} = (\sqrt{3})^5 = \sqrt{3^5} = 9\sqrt{3}$ • Convert between expressions involving negative exponents and those involving positive exponents. *Example*: $3^{-2} = \frac{1}{3^2} = \frac{1}{9}$; $\frac{2^{-3}}{7^{-1}} = \frac{7}{2^3} = \frac{7}{8}$

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O2. Complex numbers	
a. Represent complex numbers in the form *a+bi,* where *a* and *b* are real; simplify powers of pure imaginary numbers.	• Every real number, a, is a complex number because it can be expressed as $a+0i$. • Represent the square root of a negative number in the form *bi*, where *b* is real; simplify powers of pure imaginary numbers. *Example*: $\sqrt{-8}=2i\sqrt{2}$ *Example*: $\sqrt{-256}=16i$ *Example*: $i^5=i$
b. Perform operations on the set of complex numbers.	• Add, subtract, and multiply complex numbers using arithmetic rules . • Divide complex numbers using conjugates, $a+bi$ and $a-bi$. *Example*: $\frac{(5+4i)}{3-2i}=\frac{(5+4i)}{(3-2i)}\cdot\frac{(3+2i)}{(3+2i)}=\frac{15+22i+8i^2}{9-4i^2}=\frac{7+22i}{13}=\frac{7}{13}+\frac{22}{13}i$
O3. Algebraic expressions	
a. Convert between and among radical and exponential forms of algebraic expressions.	*Example*: $x^{\frac{5}{4}}=\sqrt[4]{x^5}=x\sqrt[4]{x}$ *Example*: $\sqrt{25a^6b^4}=\lvert 25\rvert^{\frac{1}{2}}\lvert a\rvert^{\frac{6}{2}}b^{\frac{4}{2}}=\lvert 5\rvert\lvert a\rvert^3b^2$
b. Simplify and perform operations on radical algebraic expressions.	*Example*: $\sqrt{x^2+6x+9}=\sqrt{(x+3)^2}=\lvert x+3\rvert$

c. Apply the laws of exponents to algebraic expressions, including those involving rational and negative exponents, to order and rewrite them in alternative forms.	*Example*: $5a^4 \cdot 3a^3 = 15a^{(4+3)} = 15a^7$, $\frac{a^4}{a^3} = a^{(4-3)} = a$, $\left(a^4\right)^3 = a^{(4\bullet 3)} = a^{12}$ *Example*: $y^{-2} = \frac{1}{y^2}$, $z^{\frac{2}{3}} = \sqrt[3]{z^2} = \left(\sqrt[3]{z}\right)^2$ *Example*: $\frac{r^{-3}}{t^{-2}} = \frac{t^2}{r^3}$, $\frac{a^{-4}}{b^3} = \frac{1}{a^4b^3}$, $(a^3b^5)^2 = a^6b^{10}$
d. Perform operations on polynomial expressions.	• Division may be performed using factoring. *Assessment Limitations:* Multiplication should be limited to multiplication of, at most, a binomial by a trinomial. In division, the divisor should be limited to a linear or factorable quadratic polynomial.
e. Perform operations on rational expressions, including complex fractions.	*Example:* $\frac{(a+b)}{\left(\frac{1}{a}+\frac{1}{b}\right)} \div ab = \frac{(a+b)}{\frac{(b+a)}{ab}} \div ab = (a+b) \cdot \frac{ab}{(b+a)} \div ab = ab \div ab = 1$. *Assessment Limitations:* Expressions should be limited to linear and factorable quadratic denominators. Complex fractions should be limited to simple fractions in numerators and denominators.
f. Identify or write equivalent algebraic expressions in one or more variables to extract information.	*Example*: The expression, *C* + 0.07*C*, represents the cost of an item plus sales tax, while 1.07*C* is an equivalent expression that can be used to simplify calculations of the total cost. *Example*: $x - \frac{y^2}{x}$ can be rewritten as $\frac{x^2 - y^2}{x}$ *Example:* The expression, 0.81*x*-16.2, represents the final amount paid for an item of original price *x*. If sales tax is 8%, then the equivalent expression 1.08(0.75*x*-15) shows that $15 was taken off the price after a 25% discount was taken.

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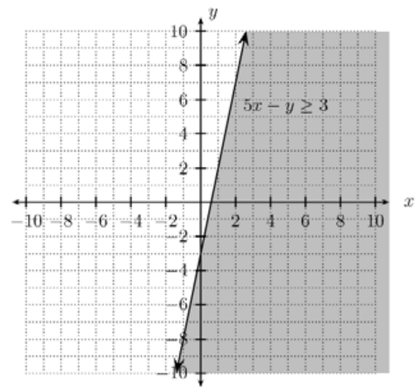
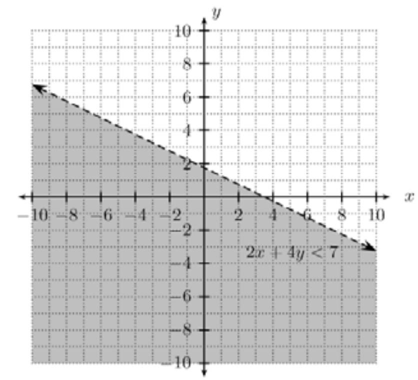
Core: Equations and Inequalities
Priority: 20%

Successful students will be able to solve and graph the solution sets of equations and inequalities and systems of linear equations and inequalities. The types of equations are to include linear, linear absolute value, quadratic, exponential, rational, radical, and higher order polynomials; the types of inequalities are to include linear and quadratic. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and, where appropriate, solve contextual problems. In contextual problems students will be required to graph and interpret their solutions in terms of the context. (Contextual test items will be limited to inequalities, systems of equations and inequalities, and those equations that do not represent a function.)

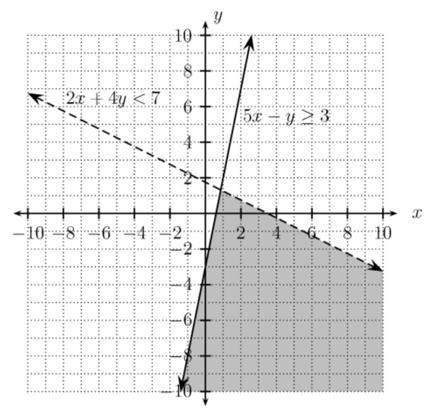
Content Benchmarks	Explanatory Comments and Examples
E1. Linear equations and inequalities	
a. Solve equations and inequalities involving the absolute value of a linear expression.	*Example*: $\lvert x-6\rvert \le 8 \Leftrightarrow (x-6)\le 8 \quad and \quad -(x-6)\le 8$ $\Leftrightarrow x\le 14 \quad and \quad x\ge -2 \quad \Leftrightarrow \quad -2\le x\le 14$
b. Express and solve systems of linear equations in three variables with and without the use of technology.	*Example*: Solve the system $2x+z=11$, $x+2y=7$, $2x-4y+z=3$ OR $x-y+z=6$, $2x+2y-z=1$, $3x+4y+3z=4$ *Assessment Limitation:* Systems in three variables should be limited to those with integer solutions and small integral coefficients.
c. Solve systems of linear inequalities in two variables and graph the solution set.	*Example*: Graph the solution set of the following system. $3x-2y<7, \quad x\ge 0, \quad y\le 0$ • Recognize that the graphic solution of a linear inequality is either an open or closed half plane. *Example*: Graphs (*a*) and (*b*) illustrate that the graph of a linear inequality is a half plane. Graph (*c*) illustrates a solution to the question: What is the set of points (x, y) that satisfies both $5x-y\ge 3$ and $2x+4y<7$?



(*a*)



(*b*)



(*c*)

- For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

d. Recognize and solve problems that can be represented by single variable linear equations or inequalities or systems of linear equations or inequalities involving two or more variables. Interpret the solution(s) in terms of the context of the problem.	• This includes using and interpreting appropriate units of measurement and precision for the given application. • Common problems are those that involve time/rate/distance, percentage increase or decrease, ratio and proportion, mixture problems and break-even problems.

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- For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

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<tr><td colspan="2">E2. Nonlinear equations and inequalities</td></tr>
<tr><td>a. Solve single-variable quadratic, exponential, rational, radical, and factorable higher-order polynomial equations over the set of real numbers, including quadratic equations involving absolute value.</td><td>

- Use information gathered from a polynomial equation to determine the number and nature of its solutions:
  - The number of roots, real and complex and counted by multiplicity for a polynomial is equal to its degree.
  - Some polynomial equations, including quadratics, may have no real solutions.
  - All complex solutions come in pairs (conjugates) making it necessary that all odd polynomial functions have at least one real solution.
- Explain the relationship between the number of real (and complex) solutions and the graph of a polynomial equation.
- Solve equations and inequalities numerically, graphically, and algebraically, with and without the use of technology, making connections between solution strategies.
- Solve power equations with integer exponents, $ax^n = b$ algebraically, graphically, and using technology.
  Power equations can represent area or volume; polynomial equations can represent projectile height, profit, or revenue.
- Solve radical equations numerically, algebraically, and graphically, with and without the use of technology.
- Use the factored form of a polynomial to determine its real roots.
- Consider domain restrictions (asymptotes or undefined values) when finding solutions of rational and radical equations.
- Know which operations on an equation may produce an equation with the same solutions, and which produce an equation with fewer or more solutions (or extraneous solutions).

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b. Solve single variable quadratic equations and inequalities over the complex numbers; graph real solution sets on a number line.	• Solve quadratic equations and inequalities using factoring, completing the square, and the quadratic formula. • Use a calculator to approximate the solutions of a quadratic equation related to an inequality and as an aid in graphing. • Select and explain a method of solution (e.g., exact vs. approximate) that is effective and appropriate for a given problem. • Determine a single variable quadratic equation given its solutions. • Recognize that complex solutions come in conjugate pairs of the form $a + bi$ and $a - bi$ when the coefficients of a quadratic equation are real numbers.
c. Use the discriminant, $D = b^2 - 4ac$, to determine the nature of the solutions of the equation $ax^2 + bx + c = 0$.	• Describe how the discriminant, $D = b^2 - 4ac$, indicates the nature of the solutions of the equation $ax^2 + bx + c = 0$. *The solutions are real and distinct if D > 0; real and equal if D = 0; and complex conjugates if D < 0.*
d. Graph the solution set of a two-variable quadratic inequality in the coordinate plane.	• Students might also be asked to write or identify the quadratic inequality from the graph. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

e. Rewrite nonlinear equations and inequalities to express them in multiple forms in order to facilitate finding a solution set or to extract information about the relationships or graphs indicated.	• Inequalities and equations that do not represent functions may be included. (*e.g.* horizontal parabolas) *Example*: Explain how to find the *x*-intercept for the equation $x = y^2 - 6y + 8$. *Example*: For $y + 1 \geq (x - 3)^2$ find the vertex and sketch a graph. • Inequalities and equations in one variable will be included. *Example*: $x^2 + 8x + 7 = 0 \Leftrightarrow (x + 7)(x + 1) = 0 \Leftrightarrow x = -7$ or $x = -1$ • Rewriting or solving a formula in several variables for one variable in terms of the others will be included. *Example*: Solve $V = \frac{1}{3}\pi r^2 h$ for r or h. *Example*: Solve $A = \pi r^2 + 2\pi r h$ for r or h.

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Core: Polynomial and Rational Functions
Priority: 30%

Successful students will be able to use tables, graphs, verbal statements and symbols to represent and analyze quadratic, rational, and higher-order polynomial functions. They will be able to recognize and solve problems that can be modeled using these functions. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
P1. Quadratic functions	
a. Determine key characteristics of quadratic functions and their graphs.	• Key characteristics include domain and range, vertex, minimum/maximum, intercepts, axis of symmetry, and end behavior. • Recognize that except when *a* = 0, the graph of $f(x)=ax^2+bx+c$ is a parabolic curve that always crosses the y-axis but may or may not cross the *x*-axis, and which opens up when *a* > 0 and down when *a* < 0. • Recognize the relationship between the intercepts and the factors of a quadratic function. • Recognize $x=\frac{-b}{2a}$ as a way of determining the x-coordinate of the vertex for a parabola in the form $y=ax^2+bx+c$. • Recognize the relationship between complex solutions of a quadratic equation and the characteristics and position of its graph. (i.e. If the graph of a function has no x-intercepts, then the function has no real roots.) *Example*: Use different forms of the function to extract information: $y=x^2-6x+8$ makes the y-intercept obvious, $y=(x-2)(x-4)$ provides access to the zeros, and $y=(x-3)^2-1$ makes it easy to find the vertex and sketch the graph.

b. Represent quadratic functions using tables, graphs, verbal statements, and equations. Translate among these representations.	• For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
c. Describe and represent the effect that changes in the parameters of a quadratic function have on the shape and position of its graph.	• Transform the parent quadratic function, $y = x^2$, to translate, reflect, rotate, stretch or compress the graph. • Investigate the changes that occur when the coefficients, including the constant are changed in a quadratic function in the form, $f(x) = ax^2 + bx + c$. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
d. Recognize, express, and solve problems that can be modeled using quadratic functions. Interpret their solutions in terms of the context.	• Recognize situations in which quadratic models are appropriate; create and interpret quadratic models to answer questions about those situations. • This includes using and interpreting appropriate units of measurement and precision for the given application. *Example*: Determine the height of an object above the ground t seconds after it has been thrown upward at an initial velocity of v_0 feet per second from a platform d feet above the ground. *Example*: Determine the relationship between the length of a side of a cube and its surface area. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

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P2. Higher-order polynomial and rational functions	
a. Determine key characteristics of power functions in the form $f(x)=ax^n$, $a \neq 0$, for positive integral values of *n* and their graphs.	• Recognize the basic power functions as the parent functions for many types of polynomials. For a polynomial function that results from transformations on a power function, that particular power function would be considered the parent function of that polynomial, such as $y = x^3$ is the parent function of $y = 3(x-2)^3 - 1$. *Example*: Determine the parent function from a graph of a polynomial that is the transformation of a power function (translated, stretched, compressed, reflected). • Identify the type of symmetry and relate to odd/even exponents.
b. Determine key characteristics of polynomial functions and their graphs.	• Key characteristics include domain and range, intercepts, end behavior, and degree. *Know that polynomial functions of degree n have the general form* $f(x)=a_n x^n + a_{n-1}x^{n-1} + ... + a_2 x^2 + a_1 x + a_0$ *where n is a positive integer. The degree of the polynomial function is the largest power of its terms for which the coefficient is non-zero. The leading coefficient is the coefficient of the term of highest degree.* o Identify and use the degree of a polynomial or its factors to interpret characteristics of the function or its graph. o Note that every polynomial function of odd degree has at least one real zero. o Understand the concept of the *multiplicity* of a root of a polynomial equation and its relationship to the graph of the related polynomial function.
c. Represent polynomial functions using tables, graphs, verbal statements, and equations. Translate among these representations.	• Identify or write a polynomial function of a given degree. • Decide if a given graph or table of values suggests a higher-order polynomial function.
d. Determine key characteristics of simple rational functions and their graphs.	• Key characteristics include domain and range, intercepts, types of symmetry, horizontal and vertical asymptotes, and end behavior. • Simple rational functions are those with linear, quadratic, or monomial denominators, including power functions of the form $f(x)=ax^n$ ($a \neq 0$) for negative integral values of *n*.

e. Represent simple rational functions using tables, graphs, verbal statements, and equations. Translate among these representations.	• Simple rational functions are those with linear, quadratic, or monomial denominators, including power functions of the form $f(x) = ax^n$ ($a \neq 0$) for negative integral values of *n*. *Example:* Graph $f(x) = \frac{x}{x^2 - x} + 5$. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
f. Recognize, express, and solve problems that can be modeled using polynomial and simple rational functions. Interpret their solutions in terms of the context.	• This includes using and interpreting appropriate units of measurement and precision for the given application. *Example*: The volume of a cylinder whose radius is 5 more than twice the radius, *r*, of a given sphere and whose height is twice its own radius is 6,750 π. Determine the radius of the given sphere. • Solve problems involving direct, inverse, and joint variations. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

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Core: Exponential Functions
Priority: 20%

Successful students will be able to use tables, graphs, verbal statements and symbols to represent, analyze, model, and interpret graphs of exponential functions. While some facility with the properties of logarithms may be helpful, it is not required on the core exam. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections and solve rich contextual problems.

Content Benchmarks	Explanatory Comments and Examples
X1. Exponential functions	
a. Determine key characteristics of exponential functions and their graphs.	• Key characteristics include domain and range, intercepts, and end behavior.
b. Represent exponential functions using tables, graphs, verbal statements, and equations. Represent exponential equations in multiple forms. Translate among these representations.	• Know that exponential functions have the general form $f(x) = ab^x + c,\ b > 0,\ and\ b \neq 1.$ • Distinguish between and graph exponential functions that are growth functions, such as $f(x) = 3 \cdot 2^x$ where x > 0, or decay functions, such as $f(x) = 3 \cdot 2^{-x}$ where x > 0. • Decide if a given graph or table of values represents an exponential function. *Be aware that it can be very difficult to distinguish exponential graphs from graphs of other functions, particularly polynomial functions, over small regions or particular subsets of their domains. Sometimes the context of an underlying situation can suggest a likely type of function model.* • Translate from exponential to logarithmic form and vice versa (e.g. $a^b = c \rightarrow log_a c = b$) • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

c. Describe and represent the effect that changes in the parameters of an exponential function have on the shape and position of its graph.	• Explain or illustrate the effect that changes in a parameter (*a* or *c*) or the base (*b*) have on the graph of the exponential function $f(x) = ab^x + c$. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
d. Recognize, express, and solve problems that can be modeled using exponential functions, including those where logarithms provide an efficient method of solution. Interpret their solutions in terms of the context.	• A logarithm is an exponent that depends on the base used. Logarithms provide an efficient method for solving problems with variable exponents. Limitation: Logarithms may be used to solve problems but will not be used in the text of an item. *If* $x^y = z, x > 0, x \neq 1,$ *and* $z > 0,$ *then y is the logarithm to the base x of z. The equation* $y = \log_x z$ *is one of three equivalent forms of expressing the relation* $x^y = z$ *(the other being* $x = \sqrt[y]{z}$ *).* • Exponential problems in which the student must determine the exponent may also be solved using graphing techniques. • Exponential growth functions using base *e* may be included (e.g. $A = Pe^{rt}$, or $y = Ce^{kt}$). Formulas will be provided in these cases. • This includes using and interpreting appropriate units of measurement and precision for the given application. *Example*: If a culture had 500 cells at noon and 600 cells at 1:00 PM, what is the approximate doubling time of the cell population? Approximately how many cells will there be at 4:00 PM? *Example*: If radioactive iodine 123 has a half-life of 8 days, what percentage of an original dose will remain in a patient's body twelve hours after a medical test has been performed in which iodine 123 was administered? • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

Core: Function Operations and Inverses
Priority: 15%

Successful students will be able to perform function operations of addition, subtraction, multiplication, division, and composition and to combine several functions defined over restricted domains to form a piecewise-defined function. They will be able to determine, graph and analyze the inverse of a function and use composition to determine whether two functions are inverses. There are a variety of types of test items including some that cut across the objectives in this standard and require students to make connections.

Content Benchmarks	Explanatory Comments and Examples
F1. Function operations	
a. Combine functions by addition, subtraction, multiplication, and division.	*Example*: If $f(x)=3x^4-5x^3+3$, and $g(x)=x^4-3x^3+2x^2+5$, then $f(x)-g(x)=$ $(3x^4-5x^3+3)-(x^4-3x^3+2x^2+5)$ $=(3x^4-x^4)+(-5x^3-(-3x^3))-2x^2+(3-5)$ $=2x^4-2x^3-2x^2-2$ *Example:* If $h(x) = x^4 + x^3 + 2x^2 + 6x + 4$ and $j(x) = x + 1$, then $h(x) \div j(x) = x^3 + 2x + 4,\ x \neq -1$. *Sample Solution:* Using synthetic division $\begin{array}{r\|rrrrr} -1 & 1 & 1 & 2 & 6 & 4 \\ & & -1 & 0 & -2 & -4 \\ \hline & 1 & 0 & 2 & 4 & 0 \end{array}$ *Example*: If $r(x)=x^2+3x-1$ and $s(x)=x+3$, then $r(x)\cdot s(x)=(x^2+3x-1)(x+3)=x^3+6x^2+8x-3$
b. Determine the composition of two functions, including any necessary restrictions on the domain.	*Example*: If $f(x)=3x-2$, and $g(x)=\sqrt{x+1}$ find $f(g(x))$ and $g(f(x))$. Include domain restrictions on each. *Example*: If $f(x)=\frac{3}{5}+\frac{2}{5x}$, and $g(x)=\frac{2}{5x-3}$, determine whether or not f and g are inverses and explain how you know.

F2. Inverse functions	
a. Describe the conditions under which an inverse relation is a function.	• Consider graphic conditions for an inverse relation to be a function. • Use the horizontal line test to determine whether the inverse of a function is also a function. Consider domain restrictions for existence of an inverse function. • Recognize that the inverse of a quadratic function is a function only when its domain is restricted.
b. Determine and graph the inverse relation of a function.	• Include inverses which may not be functions. *Example*: The inverse relation of $y = x^2$ is $y = \pm\sqrt{x}$. Explain why an inverse *function* would be only either the positive or the negative part of the graph. • Explain why the graphs of a function and its inverse are reflections of each other over the line *y* = *x*. • Show that when the inverse of a function is a function $f^{-1}\left(f(x)\right) = x$ and $f\left(f^{-1}(x)\right) = x$ *Example*: Determine the inverse relation for $f(x) = 3x^2 + 5$. *Example*: Determine *g(x)* when $g^{-1}(x) = 9x - 81$. • Represent the inverse of an exponential function graphically but not algebraically. *Example*: Graphically represent $f^{-1}(x)$ when $f(x) = 2^x - 7$. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

F3. Piecewise-defined functions	
a. Determine key characteristics of absolute value, step, and other piecewise-defined functions.	• Key characteristics include vertices, intercepts, end behavior, slope of linear sections, and discontinuities. • Determine the vertex, slope of each branch, intercepts, and end behavior of an absolute value graph.
b. Represent piecewise-defined functions using tables, graphs, verbal statements, and equations. Translate among these representations.	• Interpret the algebraic representation of a piecewise defined function; graph it over the appropriate domain. • Write an algebraic representation for a given piecewise defined function. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).
c. Recognize, express, and solve problems that can be modeled using absolute value, step, and other piecewise-defined functions. Interpret their solutions in terms of the context.	• This includes using and interpreting appropriate units of measurement and precision for the given application. • Applications may include postage rates, salary increases, or cellular telephone charges. • For items where a student is required to graph the solution set of an equation or function, axes and scales should be labeled. If the item is written in a context, the labels and scales must be appropriate within the context of the item, including units (e.g., dollars, seconds, etc).

ADP Algebra I and Algebra II End-of-Course Exam Notation Information

The information below is meant to inform teachers, schools, districts, and states about the notation **that students will see on the ADP Algebra I and Algebra II End-of-Course Exams.** It is not meant to exclude acceptable notation from being used in the classroom, but to let teachers know what students should expect to see on the exam. We expect students to be exposed to and use multiple forms of correct notation in their classrooms. In addition, students may answer items using any acceptable form of notation on the exam.

Notation on Both Algebra I and Algebra II End-of-Course Exams

Absolute value functions: $f(x) = -3|x+2| + 1$

Set notation: $\{-1, 0, 4\}$ for solution sets

Negative fractions and rational expressions: $-\frac{2}{3}$ $-\frac{x+1}{x}$

Monomials involving roots and exponents: $x^2 y \sqrt[4]{z^3}$
(x squared multiplied by y multiplied by the fourth root of z cubed)

Notation on Algebra II End-of-Course Exams – Core

Piecewise functions: $f(x) = \begin{cases} x-2, & x \le 0 \\ x+2, & x > 0 \end{cases}$

Greatest Integer Function: $f(x) = [x]$

Exponential functions of base e: $f(x) = 2e^{-0.024x}$

Domain restrictions: When a student is asked only to simplify a rational expression, all expressions are assumed to be defined. If a student is identifying an equivalent expression or solving equations, restrictions on the domain will be a part of the item, either in the question or the answer or both.

Composition of functions: $f(g(x))$

Variables with subscripts involving exponents: ${v_0}^2$ ${R_1}^2$

ADP Algebra II End-of-Course Exam Expectations of Geometry Knowledge

The following topics are mathematical concepts with which students entering an Algebra II curriculum should be familiar from prior mathematics courses. The high school curriculum or course sequence that a student might follow that leads them to this exam varies by state, district, and sometimes even school. Some curriculum sequences include an integrated series or geometry before or after Algebra II. Regardless of the course sequence followed, the mathematical concepts below are typically considered middle school concepts and taught before the Algebra II, or its equivalent, course(s).

Prior geometry knowledge/topics where formulas would not necessarily be provided:

Sum of the interior angles of a triangle equals 180°
Perimeter of polygons
Area of rectangles
Area of triangles (not requiring trigonometry)
Area and circumference of circles
Surface area of right prisms with rectangular or triangular bases
Volume of rectangular prisms
Pythagorean Theorem
Similar figures

Use of π:

When specified that an exact answer is required, answers should be expressed in terms of π. If not specified, answers may be expressed in terms of π, or 3.14 or 22/7 may be used as an approximation for π.