Patterns and Functions & Algebra K-8

CONTENT STANDARD #9 – PATTERNS AND FUNCTIONAL RELATIONSHIPS

Understand various types of patterns and functional relationships

Topic: Patterns & Functions

Understanding(s): *Students will understand that…*	**Essential Question(s):**
• Patterns help us to remember many facts as mathematical ideas. • Understanding patterns lead to mathematically reasonable predictions and problem solutions.	• How do generalized rules help support remembering, understanding, applying, and extending mathematical ideas?
Knowledge: *Students will know…*	**Skill(s)**: *Students will be able to…*
• Numerical patterns. • Spatial patterns. • Repeating patterns. • Growing/shrinking patterns. • Geometric patterns. • Functions (Grade 3 and up). • Reasonable predictions. • Generalized rules.	• Describe various types of patterns. • Compare and contrast various types of patterns (Grade 2 and up). • Generalize rule(s) for patterns and functions. • Use generalized rules to solve problems. • Extend patterns (what comes next, and what comes later). • Conjecture and predict the next and subsequent elements in a given pattern. • Create different types of patterns. • Represent generalized rules in a variety of forms (models, pictures, words, numerical expressions, etc.).

Grade	Reference	Benchmark
Grade 8	MA.8.9.1	Represent a variety of patterns (including recursive patterns) with tables, graphs (including graphing technology when available), words, and when possible, symbolic rules
	MA.8.9.2	Use linear relationships with two variables to solve problems
	MA.8.9.3	Identify functions as linear or nonlinear and contrast their properties from tables, graphs (including graphing technology when available), or equations
Grade 7	MA.7.9.1	Create a pattern or function for a rule given in symbolic form
	MA.7.9.2	Describe multi-step functions using words and symbols when given a table of "input" and "output" values and use the rule for the function to determine other input and output values
Grade 6	MA.6.9.1	Represent visual and numerical patterns with tables and graphs and generalize the "rule" using words and symbols
	MA.6.9.2	Describe simple one-step functions using words and symbols when given a table of "input" and "output" values
Grade 5	MA.5.9.1	Analyze patterns and functions and use generalizations to make reasonable predictions
	MA.5.9.2	Describe situations in which the relationship between two quantities vary directly or inversely
Grade 4	MA.4.9.1	Extend, create, and generalize growing and shrinking numeric and geometric patterns (including multiplication patterns)
	MA.4.9.2	Represent the relationship between quantities in a variety of forms (e.g., manipulatives, tables, pictures, symbols)
Grade 3	MA.3.9.1	Create and describe growing numerical and spatial patterns and generalize a rule for the pattern
	MA.3.9.2	Use patterns to solve problem situations involving related quantities in which one quantity changes as the other changes
	MA.3.9.3	Identify and describe patterns in a hundreds chart
Grade 2	MA.2.9.1	Describe and create addition and subtraction number patterns (e.g., [20, 17, 14, …])
	MA.2.9.2	Use different forms (e.g., concrete, pictorial, numerical) to represent the same basic pattern
	MA.2.9.3	Demonstrate and explain the difference between repeating patterns and growing patterns
Grade 1	MA.1.9.1	Extend, create, and describe repeating patterns
Grade K	MA.K.9.1	Demonstrate repeating patterns involving shapes, objects, sounds, and movements

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 8	• Determine the next three values in a given sequence of numbers, e.g., given the sequence 3, 7, 11, 15 … conclude that the next three values will be 19, 23, and 27.	• Organize the values in a given sequence using a table and/or graph, e.g., where "x-value" represents the placement in the sequence, i.e., one for the first term, two for the second term, etc., and the y-value represents the value of the term. *NOTE: Include different kinds of patterns, such as numerical, spatial, and recursive.*	• Organize the values in a given sequence using a table and/or graph and determine the recursive pattern in the sequence, e.g., given the sequence 3, 7, 11, 15 … conclude that the next number is obtained by adding 4 to the previous value.	• Organize the values in a given sequence using a table and/or graph and be able to state an explicit rule to find the value of the "nth" term either symbolically or verbally, e.g., given the sequence 3, 7, 11, 15 … conclude that the rule is y = 4x - 1, or an equivalent form, or verbally describing that you have to multiply the term number by 4 and then subtract 1.	**MA.8.9.1** **Represent a variety of patterns (including recursive patterns) with tables, graphs (including graphing technology when available), words, and when possible, symbolic rules**	• Explain how a table of values can be used to determine whether a function is linear or nonlinear. *Note: Explanation should include an example to demonstrate each.*
	• Determine values of a function for a given context, e.g., given C = 7t, where t is the number of tickets purchased and C is the total cost, find the cost for various values of t. *NOTE: Use functions of the form y = mx.*	• Explain what the value for the slope of a function represents in a given context, e.g., given C = 7t, where t is the number of tickets purchased and C is the total cost, explain that "7" represents the cost per ticket. *NOTE: Use functions of the form y = mx.*	• Explain what the values for the slope and y-intercept of a function represent in a given context, e.g., given a linear function showing the relationship between number of hours parked and total parking charge, C = 1.5h + 3, explain that 1.5 is the hourly charge and the 3 is the entry fee into the parking lot.	• Write an equation to represent a given problem situation that involves a linear relationship and use the equation to determine various values, e.g., presented with, "A parking garage charges a $3.00 entry fee plus an hourly charge of $1.50. Write a linear equation to represent the amount an individual would pay." The student creates y = 1.50x + 3 and uses the function to determine the charge for various times.	**MA.8.9.2** **Use linear relationships with two variables to solve problems**	• Describe a real or hypothetical situation where one variable directly or indirectly affects the value of a related variable, e.g., as one variable increases the other increases or as one increases the other decreases.

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	Foundational Benchmark ⟷ Approaching the					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
	• Conclude if a function is linear or nonlinear by looking at its graph (when a graph is provided).	• Conclude if a function is linear or nonlinear by plotting the values from a table and analyzing the resulting graph.	• Describe how the values in a table change, e.g., in the table below, the y-value changes by three as each x-value changes by one. *x* | y *1* | 2 *2* | 5 *3* | 8 • Student recognizes that this pattern implies a linear relationship.	• Use a table of values, its graph, and/or its equation to determine whether a function is linear or nonlinear.	**MA.8.9.3 Identify functions as linear or nonlinear and contrast their properties from tables, graphs (including graphing technology when available), or equations**	• Write an equation to represent a given problem situation that involves a linear relationship and use the equation to determine various values, e.g., presented with, "A parking garage charges a $3.00 entry fee plus an hourly charge of $1.50. Write a linear equation to represent the amount an individual would pay." The student creates y = 1.50x + 3 and uses the function to determine the charge for various given times and to determine the number of hours parked for various given charges.
Gr. 7			• Evaluate a given expression for specific values, e.g., determine the value of 3x + 1 when x = 5.	• Make a table of values for a rule given in symbolic form.	**MA.7.9.1 Create a pattern or function for a rule given in symbolic form**	

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Students will	**Students will**	**Students will**	**Students will**		**Students will**
• Use the rule to determine the "output" values for one-step functions, e.g., given the rule y = -3x, find the missing values in the table below: Input *(x)* | Output *(y)* 1.5 | 0 | -4 | $^2/_3$ |	• Use the rule to determine the "output" values for multi-step functions, e.g., given the rule y = 4x + 5, find the missing values in the table below: Input *(x)* | Output *(y)* 15 | 3 | -2 | $^2/_3$ |	• Select the appropriate rule for a given table of input and output values from a list of possible choices by using guess-and-check to verify which rule is the correct, e.g., which of the following rules is true for all pairs of values in the table below? Input (x) | Output (y) 1 | 3 2 | 7 3 | 11 4 | 15 a. 3x b. 4x-1 c. 1-4x d. 3x+1 • Use the rule to determine other "input" or "output" values for one-step functions when one of the values is known, e.g., given the rule y = 4x, find the missing values in the table below: Input *(x)* | Output *(y)* 15 | | 31	• Describe multi-step functions when given a table of input and output values by: ○ Explaining in words how to use the input value to determine the output value. ○ Writing a symbolic rule that can be used to calculate the output value when the input value is given (or vice versa). e.g., Input *(x)* | Output *(y)* 0 | 1 1 | 3 5 | 11 10 | 21 Rule (words): Double the input value then add 1 to determine the output value. Rule (symbols): y = 2x + 1. • Use the rule to determine other "input" or "output" values for multi-step functions when one of the values is known, e.g., given the rule y = 2x + 1, find the missing values in the table below: Input *(x)* | Output *(y)* 13 | | 51	**MA.7.9.2** **Describe multi-step functions using words and symbols when given a table of "input" and "output" values and use the rule for the function to determine other input and output values**	

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Gr. 6	• Describe visual OR numeric patterns in words.	• Represent visual AND numeric patterns with tables. e.g., Input (x) | Output (y) 0 | 0 1 | 2 2 | 4 3 | 6	• Represent visual and numerical patterns with tables and graphs, e.g., graph the input/output values in the table.	• Represent visual and numerical patterns with tables and graphs and generalize the "rule" using words and symbols.	**MA.6.9.1** **Represent visual and numerical patterns with tables and graphs and generalize the "rule" using words and symbols**	• Represent visual and numeric patterns with tables and graphs and generalizes a multi-step "rule" using words and symbols.
	• Complete a table of input/output values given a rule.	• Complete a table of input/output values, describes how to determine the missing value but may or may not be able to state a specific rule.	• Describe in words a simple one-step function using a generalized rule when given a table of input/output values, e.g., how to use the input value to determine the output value but may or may not be able to state the rule symbolically, e.g., Input (x) | Output (y) 0 | 0 1 | 2 2 | 4 3 | 6 Rule (words): Multiply the input value by two to determine the output value.	• Describe and represent simple one-step functions when given a table of input and output values in words, tables, graphs, and symbols, e.g., Input (x) | Output (y) 0 | 0 1 | 2 2 | 4 3 | 6 Rule (symbolic form): $y = 2x$	**MA.6.9.2** **Describe simple one-step functions using words and symbols when given a table of "input" and "output" values**	• Describe and represent multi-step functions when given a table of input and output values using words, graphs, and symbols.

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Gr. 5	• Make attempts to predict what comes next in a pattern or function based on a given generalized rule of the pattern (may or may not be correct).	• Make a reasonable prediction of what comes next in a pattern based on a given generalized rule of the pattern.	• Make a reasonable prediction of what comes next in a pattern and future possibilities using a given generalized rule for the pattern.	• Make and justify a reasonable prediction of what comes next in a pattern and future possibilities after determining a rule for the given pattern.	**MA.5.9.1 Analyze patterns and functions and use generalizations to make reasonable predictions**	
	• Identify the variables in a given relationship. • Determine if there is a relationship between two variables, e.g., (no relationship) the number of hours you sleep and the number of dogs being born.	• Describe how a change in one variable affects the other variable in a given direct or inverse relationship, e.g., (direct) in comparing grades and time spent studying, the **more** time spent studying, the **higher** the grade should be.	• Identify given situations as having a direct or inverse relationship between the two variables, e.g., given several scenarios, decide if there is either a direct or inverse relationship.	• Describe situations where the relationship between two variables varies directly. • Describe situations that shows the relationship between two variables vary inversely, e.g., [direct] a child's height increases as he gets older; [inverse] as more people enter a room, each person will get less personal space around him; [direct] as water is drained from a pond, the water depth decreases.	**MA.5.9.2 Describe situations in which the relationship between two quantities vary directly or inversely**	• Create and solve a situation where the relationships of two variables vary directly or indirectly, e.g., (direct) as one variable increases the other increases; (inverse) as one variable increases the other decreases.
Gr. 4	• Identify the trend of a given pattern, e.g., increasing, decreasing.	• Identify growing, shrinking, numeric, AND geometric patterns.	• Extend growing, shrinking, numeric, multiplicative, and geometric patterns.	• Extend and create growing, shrinking, numeric, multiplicative and geometric patterns, e.g., multiplicative pattern, what is the next number 20, 40, 80, 160 … ?.	**MA.4.9.1 Extend, create, and generalize growing and shrinking numeric and geometric patterns (including multiplication patterns)**	
	• Make and justify a	• Use manipulatives,	• Use manipulatives,	• Use manipulatives,	**MA.4.9.2**	• Select and justify the

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	reasonable prediction of what comes next and what comes later using a generalized rule for a pattern or function.	pictures, or symbols to represent the relationships between quantities. • Identify the pattern or rule in words, but may not be able to with numerical symbols.	tables, pictures OR symbols to represent the relationship between quantities for a pattern or function. • Describe the pattern or rule in words but may not be able to with numerical symbols.	tables, pictures AND symbols to represent the relationship between quantities for patterns and functions. • Describe the pattern and state a rule for a pattern or function in words.	**Represent the relationship between quantities in a variety of forms, e.g., manipulatives, tables, pictures, symbols.**	most effective and efficient representation for given patterns and functions.
Gr. 3	• Represent the same pattern using objects, pictures, or numbers.	• Create a basic growing pattern using objects, pictures, or numbers.	• Identify a growing numerical or spatial pattern by describing a generalized rule for the pattern.	• Create, describe and state a generalized rule for a growing numerical and spatial pattern.	**MA.3.9.1** **Create and describe growing numerical and spatial patterns and generalize a rule for the pattern**	
	• Attempt to identify what comes next in a growing or spatial pattern using objects, pictures, or numbers.	• Use the pattern to predict what comes next in a growing pattern with concrete, pictorial and/or numbers with some support.	• Describe a pattern, in which one quantity changes as the other changes.	• Describe and apply patterns to solve situations of related quantities where a change in one variable affects the value of a related variable, e.g., given a table: Time (hours) / Distance Traveled (miles) 0 / 0 1 / 50 2 / 100 3 / 150 Describe the pattern and use it to determine miles traveled for five hours.	**MA.3.9.2** **Use patterns to solve problem situations involving related quantities in which one quantity changes as the other changes**	
		• Recognize situations where two quantities	• Identify a growing pattern in a hundreds	• Describe various patterns in a hundreds	**MA.3.9.3**	

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		are related, and when one changes, so does the other, e.g., more candy purchased and more total amount paid; more purchase made and less money remaining.	chart and describe what comes next.	chart, e.g., growing/addition number patterns, subtraction number patterns.	**Identify and describe patterns in a hundreds chart**	
Gr. 2	• Recognize and identify a simple growing and/or decreasing pattern of objects, pictures, or numbers.	• Describe a simple growing pattern and predict what comes next in a sequence by paying attention to how each element in the pattern relate to each other.	• Describe addition and subtraction number patterns by stating which elements change and how it makes the pattern grow larger or smaller.	• Describe and create addition and subtraction number patterns using numbers, i.e., 20, 17, 14 … The subtraction pattern shows – 3.	**MA.2.9.1 Describe and create addition and subtraction number patterns (e.g., [20, 17, 14, …])**	• Use models, pictures and words to describe, extend and state a rule of-addition and subtraction patterns.
	• Represent repeating patterns by using objects, pictures and numbers. • Recognize and identify a repeating pattern of objects, pictures, or numbers.	• Represent and extend growing and/or decreasing patterns using objects, pictures, and numbers.	• Represent, extend, and create growing and/or decreasing patterns using objects, pictures, and numbers.	• Represent patterns by using a variety of forms, e.g., represent the addition pattern of "+3" using objects, pictures, and numbers.	**MA.2.9.2 Use different forms (e.g., concrete, pictorial, numerical) to represent the same basic pattern**	
		• Describe a growing pattern by paying attention to how each element in the pattern relates to each other.	• Describe repeating AND growing patterns. • Use comparison-contrast language to describe how two or	• Explain the difference between repeating and growing patterns by giving an example of each-using objects, pictures and numbers. • Use comparison-contrast language to describe the difference	**MA.2.9.3 Demonstrate and explain the difference between repeating patterns and growing patterns**	• Describe, create and explain addition and subtraction patterns using words and numbers, i.e., 20, 17, 14 … I know the subtraction pattern is -3 because 20 - 3 = 17. • Use comparison-contrast and cause-effect language to

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			more patterns are similar and different when presented repeating and growing patterns, e.g., given an example, such as: (AB, ABB, ABBB …) vs. (ABB, ABB, ABB …); student would say, "There's one more B each time in the 'growing pattern'; but, the 'repeating pattern' has (ABB) repeating over and over."	between repeating and growing patterns they have created, e.g., student creates the following patterns, (AB, ABB, ABBB …) vs. (ABB, ABB, ABB …). The student would say, "There's one more B each time in the 'growing pattern' but, the 'repeating pattern' has (ABB) repeating over and over."		describe the difference between repeating and growing patterns.
Gr. 1	• Predict what comes next in a simple repeating pattern, e.g., simple repeating pattern: Red, Blue, Red, Blue, Red … vs. (more complex pattern) Red, Blue, Yellow, Yellow, Red, Blue, Yellow, Yellow, Red … by paying attention to how each element in the pattern relates to each other. • Use one attribute of the elements to describe the repeating patterns, e.g., given the pattern red square, red square, blue square, the student will describe it as, "red, red, blue."	• Extend a repeating pattern by adding on to a given sequence. • Use some attributes of the elements to describe the repeating patterns, e.g., given the pattern red square, red square, blue square, the student will describe it as "red square, red square, blue square OR two red, one blue.	• Create a simple original repeating pattern.	• Extend, create, and describe repeating patterns. • Use all attributes of the elements to describe the repeating patterns, e.g., given the pattern: red square, red square, blue square, student will describe it as "two red squares, one blue square."	**MA.1.9.1 Extend, create, and describe repeating patterns**	• Extend, create, and describe repeating patterns where the "unit" being repeated contains five or more elements.
Gr.	• Match identical shapes, colors, etc., to the	• Recognize the elements that are being	• Recognize the elements that are being	• Recreate repeating patterns using a variety	**MA.K.9.1**	• Recreate repeating patterns using a variety

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K	elements in the given sequence. *Note: Students may need one-to-one assistance, modeling and prompting.*	repeated (but not necessarily in the right order) in a given sequence. • Reproduce simple repeating patterns using shapes, objects, sounds, OR movements in a given sequence but not necessarily in the right order.	repeated in a given sequence. • Recognize, reproduce and continue, simple repeating patterns using shapes, objects, sounds, AND movements in a given sequence.	of shapes, objects, sounds, AND movements, e.g., use colors/shapes to recreate an ABAB pattern. • Recognize, reproduce and continue, simple repeating patterns using shapes, objects, sounds, AND movements in a given sequence.	**Demonstrate repeating patterns involving shapes, objects, sounds, and movements**	of shapes, objects, sounds, AND movements, AND describes the "unit" that is repeated.
		• Continues an ABAB and AABB pattern with guided practice and redirection. • During independent practice, students may revert to ABAB pattern when doing another pattern.	• Recognize and continue simple ABAB patterns using color, shape and objects. *NOTE: Gross motor development may be a factor in not being able to do the movement patterns.*			

Patterns, Functions & Algebra: K-8

CONTENT STANDARD #10 – SYMBOLIC REPRESENTATIONS

Use symbolic forms to represent, model, and analyze mathematical situations

Topic: Numeric and Algebraic Representations

Understanding(s): *Students will understand that…*

- Numbers can be represented in many ways, and used for different purposes.
- Mathematical operations can be modeled in a variety of ways and used to describe events/situations.

Essential Question(s):

- Why do we need different forms of mathematical representation?

Knowledge: *Students will know…*

- Variables (symbols representing unknown values (Grade 4 and up).
- Expressions (open sentences).
- Equations (number sentences [all grades], fact families [Grades 5 and 6]).
- Graphs and tables used to represent and communicate patterns and functions (Grade 5 and up).
- Operations (addition, subtraction, multiplication, division).
- Inverse operations role in solving equations with/without unknowns.

Skill(s): *Students will be able to…*

- Select appropriate math problem strategies (i.e., smaller numbers, work backwards, organized lists, etc.).
- Draw and label graphs and tables (Grade 5 and up).
- Write an expression that matches the problem situation.
- Solve an equation with unknowns.
- Use inverse operations to keep balance in equations (Grade 7 and up).
- Compose story problems for given number sentences.
- Provide physical models and visual representations for equations involving any of the operations (addition, subtraction, multiplication, division).

Grade	Reference	Benchmark
Grade 8	MA.8.10.1	Translate among tables, graphs (including graphing technology when available), and equations involving linear relationships
	MA.8.10.2	Solve linear equations and inequalities with two variables using algebraic methods, manipulatives, or models
	MA.8.10.3	Use tables and graphs to represent and compare linear relationships
Grade 7	MA.7.10.1	Analyze the relationship among tables, graphs (including graphing technology when available), and equations of linear functions, paying particular attention to the meaning of intercept and slope
	MA.7.10.2	Use symbolic algebra to represent situations involving linear relationships
	MA.7.10.3	Solves linear equations and inequalities with one variable using algebraic methods, manipulatives, or models
Grade 6	MA.6.10.1	Interpret and solve problem situations involving two different variables
	MA.6.10.2	Use fact families to solve for an unknown in an open sentence
	MA.6.10.3	Evaluate algebraic expressions
Grade 5	MA.5.10.1	Use a variety of strategies to solve number sentences with unknowns
	MA.5.10.2	Model problem situations with objects or manipulatives and use representations (e.g., graphs, tables, equations) to draw conclusions
Grade 4	MA.4.10.1	Use symbols to represent unknown quantities in open sentences and determine the unknown quantities
	MA. 4.10.2	Represent the commutative, associative, and distributive properties symbolically
Grade 3	MA.3.10.1	Model situations that involve multiplication and division of whole numbers using objects/pictures and number sentences
Grade 2	MA.2.10.1	Create a word/story problem for a given number sentence
Grade 1	MA.1.10.1	Use objects, pictures, words, and number sentences to represent and solve numerical problem situations involving addition and subtraction
Grade K	MA.K.10.1	Represent simple numerical situations with objects and number sentences

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	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 8	• Determine the recursive rule that describes how the y-values change in a given table of values. *NOTE: The x-values in the table should change by increments of one.*	• Use the recursive rule (that describes how the y-values change) to determine the explicit rule that describes the relationship between corresponding x- and y-values in a given table. *NOTE: The x-values in the table should change by increments of one.*	• Compare several pairs of points to determine if there is a constant rate of change in a given table of values. *Note: The x-values in the table should change by increments of any size.*	• Use information from a table to: a) construct the graph and b) determine the equation of the line.	**MA.8.10.1** **Translate among tables, graphs (including graphing technology when available), and equations involving linear relationships**	• Explain how the slope and the y-intercept can be derived from using information provided in a table, graph, and equation.
	• Determine the coordinates of several points on the graph of a given line. *NOTE: One of the points should be the y-intercept.*	• Determine if a particular ordered pair is a solution for a given linear equation, e.g., is (3, 4) a solution for 3x + y = 7? No, because 3(3) + 4 ≠ 7.	• State the value of the slope and the coordinates of the y-intercept when given a linear equation in the form y = mx + b, e.g., given y = 4x - 3, conclude that the slope of the line is 4 and the line will cross the y-axis at (0, -3).	• Use a linear equation to: a) construct a table and b) draw its graph.		
		• Determine the constant change in the y-values (the "rise") and the constant change in the x-values (the "run") when given the graph of a line with at least four points clearly indicated.	• Determine the slope of a line by analyzing its graph.	• Use information from a graph to: a) construct a table of values and b) state the equation of the line.		
	• Solve a linear equation or inequality in one variable, e.g., given 2x + 5 > 6, the solution set is x > ½.	• Solve a linear equation or inequality in one variable when the equation or inequality has variables on both sides of the equation, e.g., solve 3x + 4 = x + 14.	• Rewrite a linear equation or inequality in two variables, when the y-variable has a coefficient of 1, so that the y-variable is expressed in terms of x, e.g., 2x + y = 9 can be re-written as y = -2x + 9.	• Rewrite a linear equation or inequality in two variables so that the y-variable is expressed in terms of x, e.g., 2x + 3y < 9 can be re-written as y < (-2/3)x + 3.	**MA.8.10.2** **Solve linear equations and inequalities with two variables using algebraic methods, manipulatives, or models**	• Rewrite a linear equation or inequality in two variables so that either variable is expressed in terms of the other, e.g., 2x + 3y = 9 can be re-written as y = (-2/3)x + 3 or x = (-3/2)y + 9/2.

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	Students will	**Students will**	**Students will**	**Students will**		**Students will**
	• Determine if the slope of a line is positive, negative, zero or undefined by looking at its graph.	• Describe the direction of the graph using the slope of a given linear equation, e.g., given y = -3x + 1, the line will go downwards from left to right. • Make a conclusion about which line has a greater slope when given the graphs of two linear equations.	• Determine the slope of a line from a given table of values.	• Compare the slopes of two linear relationships by referring to the change in values in a table, and the slant of their respective graphs.	**MA.8.10.3** **Use tables and graphs to represent and compare linear relationships**	• Compare the y-intercepts of two linear relationships by referring to their tables of values when zero is NOT given as one of the x-values, e.g., the table below does not explicitly provide the y-intercept. x | y 2 | 9 3 | 13 5 | 21
Gr. 7		• Identify the coordinates of the point where the graph of a line intersects the y-axis. • Use an understanding of slope to sketch a representation of a rate of change in a given graph, e.g., given the situation, "Andy walks at a rate of four miles per hour," and given a graph showing the line with the points for the distance walked for	• Use an understanding of the **y-intercept** to match the graph of a given linear equation with its equation when given in a list of possible equations, e.g., given the graphs of three different lines in the same coordinate plane - **each having a different y-intercept but the same slope** - and given a list of three linear equations, student can match each equation with the appropriate graph. • Use an understanding of **slope** to match the graph of a given linear equation with its equation when given in a list of possible equations, e.g., given the graphs of three different lines in the same coordinate plane, **each having a <u>distinctly</u>**	• Given an equation in the form y = mx + b, its table of values, which includes (0, b) and its graph: a) Recognize that the point where the line crosses the y-axis shows up in the table as (0, b) and in the equation as the constant term b) Recognize that the steepness of the graph of the line shows up as the coefficient of x in the equation and as the ratio of the change in y-values to the change in x-values in the table. *NOTE: Increments of x-values should be the same.*	**MA.7.10.1** **Analyze the relationship among tables, graphs (including graphing technology when available), and equations of linear functions, paying particular attention to the meaning of intercept and slope**	• Describe the advantages and disadvantages of representing linear functions in table vs. graphs vs. equations.

LEVELS OF PROGRESSION BELOW THE BENCHMARK Foundational ⟷ Approaching the Benchmark			AT THE BENCHMARK	BENCHMARK	ADVANCED
Students will	**Students will**	**Students will**	**Students will**		**Students will**
	each of the first five	**different slope, but**			
	hours, the student can sketch and label the corresponding vertical and horizontal changes from one data point to the next.	**the same y-intercept,** and given a list of three linear equations, student can match each equation with the appropriate graph.			
• Determine if the relationship represented by a graph or table is linear or not.	• Select from a given set of graphs or tables, the one that represents a linear relationship.	• Create a table or graph, given a linear relationship.	• Develop an equation representing real world (linear) relationships, e.g., a parking garage charges a $3.00 entry fee plus an hourly charge of $1.50; write a linear equation to represent the amount an individual would pay. The student writes: y = 3 + 1.50x OR y = 1.50x + 3.	**MA.7.10.2** **Use symbolic algebra to represent situations involving linear relationships**	
	• Replaces variables with given values in simple mathematical sentences and compute the result, e.g., given the expression 4a + 1, the student can determine the value of the expression when a = 2, a = 3, a = 15, etc.	• Identify the value of an unknown variable in a simple mathematical sentence given the statement 3 × ◯ = 15 or 3 × ◯ < 15. The student can reason what value(s) must go in the oval to make the sentence a true statement.	• Determine the solution set for linear equations and inequalities in one variable.	**MA.7.10.3** **Solves linear equations and inequalities with one variable using algebraic methods, manipulatives, or models**	

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	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 6		• Identify the two variables in the problem.	• Solve problems by describing and modeling situations with two unknowns (using physical materials, words, and/or visual representations).	• Solve "routine and non-routine" problems with two unknowns by describing and modeling (using physical materials, words, visual, numeric, and/or **symbolic [preferred]** representations) problem situations, e.g., give the possible dimensions of a rectangle with a perimeter of 24 units, so 2L + 2W = 24. *NOTE: Problems with two unknowns typically have more than one possible solution.*	**MA.6.10.1** **Interpret and solve problem situations involving two different variables**	
		• Use a guess and check method to solve for a variable.	• Identify the members of the appropriate fact family using open sentences, e.g., a + 3 = 5 3 + a = 5 5 – a = 3 5 – 3 = a	• Find the value of the missing variable by using the correct member of the fact family, e.g., *b* + 19 = 36 *b* = 36 – 19 *b* = 17	**MA.6.10.2** **Use fact families to solve for an unknown in an open sentence**	
		• Substitute values in a simple expression and simplify, e.g., when x = 3 x + 15 becomes 3 + 15 = 18.	• Substitute values in an expression involving more than one operation; has difficulty simplifying.	• Substitute values in an expression involving more than one operation, and simplify, e.g., when x = 3, 2x + x + 15 becomes 2(3) + 3 + 15 6 + 3 + 15 24	**MA.6.10.3** **Evaluate algebraic expressions**	

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	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 5		• Identify strategies such as fact families, working backwards, and guess and check, but applies inconsistently.	• Use one or more appropriate strategies to solve number sentences with unknowns.	• Use a variety of appropriate strategies, i.e., diagram, guess and check, fact families, and working backwards to solve number sentences with unknowns.	**MA.5.10.1** **Use a variety of strategies to solve number sentences with unknowns**	
		• Model problem situations and brainstorms possible conclusions.	• Model problem situations and/or use representation to ineffectively draw conclusions.	• Model problem situations by strategically using objects, manipulatives, and/or representations to draw appropriate conclusions, e.g., conjectures, possible problem solutions, etc.	**MA.5.10.2** **Model problem situations with objects or manipulatives and use representations (e.g., graphs, tables, equations) to draw conclusions**	
Gr. 4	• Identify what is unknown, i.e., what needs to be found in a given situation.	• Recognize the open sentence from a given list to represent a situation.	• Use symbols to write an open sentence to describe a particular situation.	• Use symbols to write an open sentence to represent a situation. • Determine the value of the unknown in a situation.	**MA.4.10.1** **Use symbols to represent unknown quantities in open sentences and determine the unknown quantities**	• Use manipulatives, drawings or numbers to show that subtraction and division are not commutative or associative. Counter example: $3 - 1 \neq 1 - 3$ $4 \div 2 \neq 2 \div 4$)
	• Use manipulatives, drawings or numbers to show how products remain the same regardless of the order in which factors are combined.	• Verify the distributive property by substituting or comparing values, e.g., given the statement $5(3 + 4) = 5 \times 3 + 5 \times 4$, show that the value of both sides of the equation is 35.	• Illustrate the distributive property using the area model.	• Represent the commutative, associative, and distributive properties symbolically.	**MA.4.10.2** **Represent the commutative, associative and distributive properties symbolically**	• Use manipulatives, drawings or numbers to show that subtraction and division are not commutative or associative. Counter example: $3 - 1 \neq 1 - 3$ $4 \div 2 \neq 2 \div 4$)

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	Students will	Students will	Students will	Students will		Students will

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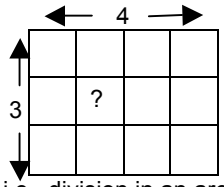
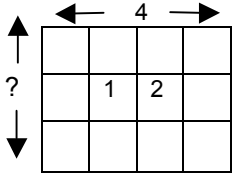
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 3			• Use manipulatives, pictures, OR number sentences to represent multiplication and division with whole numbers (i.e., multiplication as repeated addition, e.g., 5 x 3 = 5 + 5 + 5; i.e., division as repeated subtraction, e.g., 20 ÷ 5 = 20 - 5 - 5 - 5 - 5 = 0, therefore, 20 ÷ 5 = 4 since four 5s can be subtracted for 20; i.e., multiplication in an area model, e.g., given the linear units for the length and width, one can find the area in square units;  i.e., division in an area model, e.g., given either the linear units for length or width and the area in square units, one can find the missing length or width measurement, e.g., 	• Use manipulatives, pictures AND number sentences (i.e., equations, to represent multiplication and division of whole number story problems or situations).	**MA.3.10.1** **Model situations that involve multiplication and division of whole numbers using objects/pictures and number sentences**	• Use manipulatives, pictures, and number sentences to represent multiplication and division whole number situations using more than one operation in the number sentence such as (20 + 5) x 2 = ___.

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	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 2	• Use manipulatives to represent a pre-composed word/story problem.	• Match a pre-composed word/story problem to a given number sentence.	• Identify an appropriate pre-composed word/story problem for a given number sentence, e.g., select the answer from a given list.	• Creates an oral and/or written word/story problem for a given number sentence.	**MA.2.10.1** **Create a word/story problem for a given number sentence**	• Composes a word/story problem independently and represents the problem with an appropriate number sentence.
Gr. 1		• Represent addition of whole number situations by using manipulatives, pictures, OR number sentences.	• Represent addition and subtraction of whole number situations by using manipulatives, pictures, OR number sentences. • Write and explain simple mathematical statements, such as 7 + __ = 8 or ___ - 8 = 10.	• Represent addition and subtraction of whole number situation by using manipulatives, pictures, AND number sentences.	**MA.1.10.1** **Use objects, pictures, words, and number sentences to represent and solve numerical problem situations involving addition and subtraction**	• Use manipulatives, pictures, and number sentences to represent addition and subtraction whole number situations using more than one operation in the number sentence such as 10 = 20 - ___ + 2.
Gr. K	• Use manipulatives to represent numerical situations, e.g., show me five.	• Use manipulatives to represent numerical situations AND are able to identify numerals. • Identify the +, -, and = symbols/signs.	• Use manipulatives to represent numerical situations and attempts to write simple numbers sentences. • Show understanding of +, -, = by explaining or showing what each symbol/sign means.	• Use manipulatives to represent numerical situations and writes simple number sentences (equations), such as 6 = 5 + 1 or 10 - 3 = 7. • Use the +, - and = signs appropriately in the right places in a number sentence.	**MA.K.10.1** **Represent simple numerical situations with objects and number sentences**	• Use manipulatives to represent numerical situations and writes simple number sentences (equations), such as 7 + __ = 8 or ___ - 8 = 10 to model/represent numerical situations.

Patterns, Functions & Algebra: K-8

CONTENT STANDARD #10 – SYMBOLIC REPRESENTATIONS

Use symbolic forms to represent, model, and analyze mathematical situations

Topic: Rates of Change

Understanding(s): *Students will understand that…*

- Motion/change can be represented as tables, graphs, in symbols and words.

Essential Question(s):

- How does one use symbolic forms to represent, model, and analyze mathematical situations dealing with motion and change?

Knowledge: *Students will know…*

- Relationship between number patterns and graph shapes.
- Constant and varying rates of change Direct and inverse variation.
- Slope.
- Slope as rate of change (Grade 8).

Skill(s): *Students will be able to…*

- Translate between graphs, tables, symbols, and words.
- Describe constant and varying rates of change visually, numerically, and verbally.
- Finding slopes (Grade 8).

Grade	Reference	Benchmark
Grade 8	MA.8.10.4	Use the slope of a line to describe a constant rate of change
Grade 7		<no benchmark for this topic at this grade level>
Grade 6		<no benchmark for this topic at this grade level>
Grade 5	MA.5.10.3	Describe situations with constant or varying rates (e.g., miles per hour, items per box)
Grade 4	MA.4.10.3	Describe the rate of change numerically and verbally based on data recorded in a table or graph
Grade 3	MA.3.10.2	Identify situations involving change and describe the change numerically and verbally
Grade 2		<no benchmark for this topic at this grade level>
Grade 1		<no benchmark for this topic at this grade level>
Grade K		<no benchmark for this topic at this grade level>

	LEVELS OF PROGRESSION FOR BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 8			• Relate the slope to a constant rate of change for a given equation in the form **y = mx**, e.g., in y = 3x, the slope is 3 which means that as x increases by 1, y increases by 3. *NOTE: The equation that is given should be presented in two ways, sometimes **with** a provided context and sometimes context-free.*	• Relate the slope to a constant rate of change for a given equation in the form y **= mx +b**, e.g., in y = 3x + 4, the slope is 3 which means that as x increases by 1, y increases by 3. *NOTE: The equation that is given should be presented in two ways, sometimes **with** a provided context and sometimes context-free.*	**MA.8.10.1** **Use the slope of a line to describe a constant rate of change**	• Use the slope of a line flexibly to describe implicit rates of change, e.g., if given y = (2/3)x + 4, the student can state that the slope of 2/3 means that "as x increases by 1, y increases by 2/3," but it also implies that "as x increases by 6, y increases by 4" or "as x increases by 1-1/2, y increases by 1."
Gr. 7					**<no benchmarks for this grade level>**	
Gr. 6					**<no benchmarks for this grade level>**	
Gr. 5			• Identify situations as having constant or varying rates of change.	• Describe situations with constant or varying rates of change, e.g., miles per hour, plant growth over time varying.	**MA.5.10.3** **Describe situations with constant or varying rates e.g., miles per hour, items per box**	• Create a graph using appropriate conventions of graphing and explain how it reflects changes in rate. • Create a situation that reflects rates of change.
Gr. 4	• Identify situations involving rate of change from a given set of examples.	• Give an example of a rate of change, e.g., miles per hour.	• Use words OR equations to describe rates of change for data sets given in a table or graph.	• Use words AND equations to describe rates of change for data sets given in a table or graph, i.e., number of drips from a tap per minute.	**MA.4.10.3** **Describe the rate of change numerically and verbally based on data recorded in a table or graph**	

	LEVELS OF PROGRESSION FOR BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Benchmark ⟷ Approaching the					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 3	• Describe a series of objects, pictures, or numbers as increasing, staying the same, or decreasing.	• Describe how a series of objects, pictures, or numbers are increasing, staying the same, or decreasing.	• Determine whether or not change occurs in a given situation, and describe the change in words OR numbers.	• Determine whether or not change occurs in a given situation, and describe the change in words and numbers.	**MA.3.10.2 Identify situations involving change and describe the change numerically and verbally**	• Create and explain a quantitative change using words and equations.
Gr. 2					**<no benchmarks for this grade level>**	
Gr. 1					**<no benchmarks for this grade level>**	
Gr. K					**<no benchmarks for this grade level>**	