Numbers & Operations K-8

CONTENT STANDARD #1 - NUMBER SENSE

Understand numbers, ways of representing numbers, relationships among numbers, and number systems **Topic: NUMBERS AND NUMBER SYSTEMS**

Understanding(s): *Students will understand that…*	**Essential Question(s):**
• Numbers can be represented in many ways, and used for different purposes. • Equivalent representations for the same value are used for different purposes. • Understanding number theory concepts helps to represent numbers in different ways.	• How does one determine the most appropriate representation of a number in any given situation? • How does one apply number theory concepts to solve problems?
Knowledge: *Students will know…* • Even/odd numbers. • Zero. • Place value of whole numbers and decimals. • Unit fractions. • Factors of a number. • Multiples of a number. • Prime/composite numbers. • Rational/irrational numbers. • Percents. • Ratios. • Common factors and greatest common factor. • Common multiples and least common multiple. • Integers. • Absolute value. • Square root/cube root. • Scientific notation. • Numerals (Digits 0 – 9) vs. numbers (how many you have).	**Skill(s)**: *Students will be able to…* • Recall counting (whole) numbers in sequence (i.e., 1, 2, 3, 4, 5, …). • Skip count or counting on in sequence starting any number larger than one (e.g., 5, 6, 7... OR 12, 13, 14…etc.). • Provide equivalent forms for whole numbers, fractions, decimals, and percents. • Compare, order, and solve number problems (whole numbers, fractions, decimals, percents, integers, square/square roots, and cubes/cube roots). • Use number theory concepts (greatest common factors, least common multiples, prime factorization, and divisibility). • Represent large numbers in scientific notation. • Represent, identify, and use place values. • Use ratios and proportions to represent a relationship between two quantities. • Categorize numbers, e.g., even/odd, rational/irrational, prime/composite, positive/negative.

Grade	Reference	Benchmark
Grade 8	MA.8.1.1	Identify situations represented by square roots and cube roots
	MA.8.1.2	Compare and order rational numbers and square roots
	MA.8.1.3	Use ratios and proportions to represent the relationship between two quantities
Grade 7	MA. 7.1.1	Solve problems using fractions, decimals, and percents
	MA. 7.1.2	Identify situations that require the use of large numbers and represent them using scientific notation
	MA. 7.1.3	Describe and solve situations represented by integers and absolute value
	MA. 7.1.4	Apply number theory concepts to solve problems
Grade 6	MA.6.1.1	Compare and order fractions, decimals, and percents
	MA.6.1.2	Explain and give examples of number theory concepts (e.g., prime factorization, common factors, greatest common factor, common multiples, least common multiple, divisibility)

Grade	Reference	Benchmark
Grade 5	MA.5.1.1	Represent percent and ratio using pictures or objects
	MA.5.1.2	Use equivalent forms of whole numbers, fractions, ratios, decimals, and percents to solve problems
	MA.5.1.3	Use models, benchmarks, and equivalent forms to judge the size of fractions and order them
Grade 4	MA.4.1.1	Identify place value from ten-thousandths to millions
	MA.4.1.2	Identify and list factors, multiples, prime numbers, and composite numbers
	MA.4.1.3	Identify equivalent forms of commonly used fractions and decimals
Grade 3	MA.3.1.1	Represent place value from hundredths to ten-thousands flexibly
	MA.3.1.2	Categorize and justify a number as being odd or even
	MA.3.1.3	Compare and order fractions with denominators up to 12 (e.g., greater than, less than, equal)
	MA.3.1.4	Use fractions with denominators up to 12 to solve problems
Grade 2	MA.2.1.1	Represent whole numbers up to 1000 in flexible ways (e.g., relating, composing, and decomposing numbers), including the use of tens and hundreds as units
	MA.2.1.2	Compare whole numbers up to 1000 using words (e.g., greater than, less than, equal to)
	MA.2.1.3	Represent fractions with denominators no larger than ten using pictures, numbers, words, or models
Grade 1	MA.1.1.1	Count whole numbers up to 100 in a variety of ways (e.g., skip counts by 2's, 5's, 10's)
	MA.1.1.2	Identify representations of simple fractions (e.g., one-half, one-third, one-fourth)
	MA.1.1.3	Represent whole numbers up to 100 in flexible ways (e.g., relating, composing, and decomposing numbers), including the use of tens as a unit
Grade K	MA.K.1.1	Count and compare groups of objects up to 30 according to the number of objects in each group
	MA.K.1.2	Represent whole numbers up to 30 in flexible ways (e.g., relating, composing, and decomposing numbers)

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 8			• Select a situation that requires the use of square roots.	• Select a situation that requires the use of square roots and cube roots.	**MA.8.1.1 Identify situations represented by square roots and cube roots**	• Create a situation that requires the use of square roots or cube roots.
	• Compare a rational number to another rational number and state which one is greater than (or less than) the other.	• Make the following comparisons: ○ compare a rational number to another rational number and state which one is greater than (or less than) the other. ○ compare a square root to another square root and state which one is greater than (or less than) the other.	• Make the following comparisons: ○ compare a rational number to another rational number and state which one is greater than (or less than) the other. ○ compare a square root to another square root and state which one is greater than (or less than) the other. ○ compare a square root to a rational number and state which one is greater than (or less than) the other.	• Order a given set of rational numbers and square roots from greatest to least (or least to greatest).	**MA.8.1.2 Compare and order rational numbers and square roots**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK Foundational Benchmark ⟷ Approaching the			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			• Identify related quantities that can be used to set up a proportion but has difficulty correctly placing them in the proportion.	• Represent a given situation with a ratio. e.g., in a classroom with 12 boys and 12 girls, the ratio of boys to girls is 12:12 or 1:1; the ratio of boys to the total number of students is 12:24 or 1:2. • Represent a given situation with a proportion when an unknown quantity needs to be determined based on a known ratio, e.g., if a student is 5 feet tall and casts a 3-foot shadow, how tall is the flag pole that casts an 18-foot shadow? The student can set up the proportion: 5/3 = x/18).	**MA.8.1.3** **Use ratios and proportions to represent the relationship between two quantities**	
Gr. 7		• Solve problem situations that involve computations with decimals.	• Solve problem situations that involve computations with decimals and percents.	• Solve problem situations that involve computations with fractions, decimals, and percents.	**MA.7.1.1** **Solve problems using fractions, decimals, and percents**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
		• Recognize the scientific notation format of a number, i.e., n x 10p where 1< n < 10 and p is any integer. *NOTE: Students should also recognize the calculator format of scientific notation*	• Convert numbers in scientific notation to standard notation. • Convert numbers in standard notation to scientific notation.	• Select a situation that requires the use of large numbers. • Convert the large numbers in the selected situation from standard notation to scientific notation.	**MA.7.1.2** **Identify situations that require the use of large numbers and represent them using scientific notation**	• Create a problem requiring the use of large numbers. • Solve the problem using the scientific notation form of the large numbers.
		• Recognize the absolute value symbol and simplify an absolute value term, e.g., | -3 | = 3.	• Simplify a given numeric expression involving absolute values and integers.	• Describe a situation (create a problem) that involves integers and then solve the problem. • Describe a situation (create a problem) that involves absolute value, e.g., the difference in distance can be represented as an absolute value, and then solve the problem.	**MA. 7.1.3** **Describe and solve situations represented by integers and absolute value**	
			• Identify situations that involve number theory concepts.	• Solve problem situations that involve applying number theory concepts such as common factors, common multiples, prime factorization, greatest common factors, least common multiples, or divisibility, e.g., John has a particular number of marbles such that when he tries to group them in twos there is one leftover, when he	**MA. 7.1.4** **Apply number theory concepts to solve problems**	• Create a situation that involves number theory concepts and solve the problem.

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				tries to group them in threes there is one leftover, and when he tries to group them in fours there is one leftover, but when he groups them in fives, there are no marbles leftover. How many marbles could John have?		
Gr. 6		• Compare and order percents.	• Compare and order decimals and percents.	• Choose and carry out an appropriate strategy to: ○ compare a fraction to a decimal or percent and state which one is greater than (or less than) the other. ○ compare a decimal to a fraction or percent and state which one is greater than (or less than) the other. ○ compare a percent to a fraction or decimal and state which one is greater than (or less than) the other. ○ order a given set of fractions, decimals, and percents from greatest to least (or least to greatest).	**MA.6.1.1 Compare and order fractions, decimals, and percents**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
	• Explain and give examples of divisibility.	• Explain and give examples of prime factorization.	• Explain and give examples of least common multiples and greatest common factors.	• Explain how to determine the following for any given number(s), and support the explanation with a specific example: ○ prime factorization. ○ common factors. ○ greatest common factor. ○ common multiples. ○ least common multiple. ○ divisibility (by 2, 3, 4, 5, 6, 9, 10).	**MA.6.1.2** **Explain and give examples of number theory concepts (e.g., prime factorization, common factors, greatest common factor, common multiples, least common multiple, divisibility)**	
Gr. 5		• Match percents and ratios with picture or objects representation.	• Relate percents to decimals, i.e., percent means "per hundred" or "hundredths" and fractions, e.g., 5% is 5/100.	• Demonstrate an understanding of percent by representing percents with pictures or objects. • Demonstrate an understanding of ratio by representing ratios using pictures or objects. *NOTE: There are two types of ratios –* ***part-to-whole*** *(three dogs out of a total of seven cats and dogs can be represented as 3/7, 3:7, or 3 to 7);* ***part-to-part*** *(three dogs to four cats can be represented as ¾, 3:4, or 3 to 4).*	**MA.5.1.1** **Represent percent and ratio using pictures or objects**	
			• Match equivalent expressions between	• Solve a problem by substituting a whole	**MA.5.1.2** **Use equivalent forms**	

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	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			fractions, decimals, percents, and whole numbers, e.g., matches 25% of 8 to ¼ of 8 or 8 divided by 4, or 8 multiplied by 0.25.	number, fraction, ratio, decimal, or percent with an equivalent form that aids in the solution process, e.g., to find 25% of 8, the student uses ¼ in place of 25%, or multiplies by 0.25.	**of whole numbers, fractions, ratios, decimals, and percents to solve problems**	
		• Use models (fraction strips, pattern block relationships, equally shared groups of objects, etc.) to judge the size of fractions to order them from smallest to largest and vice-versa.	• Use benchmarks (0, ¼, ½, ¾, 1 on a number line or U.S. customary measurement ruler) to judge the size of fractions to order them from smallest to largest and vice-versa.	• Use models (fraction strips, pattern block relationships, equally shared groups of objects, etc.), benchmarks (0, ¼, ½, ¾, 1 on a number line or U.S. customary measurement ruler), and equivalent forms (1/2 = 2/4 = 50/100, 0.5, etc.) to judge the size of fractions to order them from smallest to largest and vice-versa.	**MA.5.1.3 Use models, benchmarks, and equivalent forms to judge the size of fractions and order them**	
Gr. 4			• Identify the digit in a given place value position of a number, e.g., the student states that 5 is in the millions	• Identify the value of any given digit from the ten-thousandths through millions. e.g., student states that 5	**MA.4.1.1 Identify place value from ten-thousandths to millions**	

LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
Foundational ⟷ Approaching the Benchmark					
Students will	**Students will**	**Students will**	**Students will**		**Students will**
		place for 35,200,111; the student states that 7 is in the ten-thousandths place in 543.2107.	represents 5 millions in 35,200,111; the student states that 7 represents 7 ten-thousandths in 543.2107. • Read a decimal number from ten-thousandths to millions, e.g., 1.234 is read as one and two-hundred thirty-four thousandths. *NOTE: While the benchmark does not require students to be able to "read" a number, it is a foundational requirement for subsequent concepts and skills.*		
• Define and give an example of factors and multiples. *NOTE: Given a multiplication basic fact, such as 3 x 4 = 12, 3 and 4 are the factors and 12 is the multiple.*	• Describe the difference between factors and multiples.	• Identify which given numbers are the factors of a given number, e.g., which of these are factors of 20? (2, 4, 6, 8, 10)	• Identify and list the factors of a given number.	**MA.4.1.2** **Identify and list factors, multiples, prime numbers, and composite numbers**	• Justify in a variety of ways that the list of factors is complete.
		• Identify which given numbers are the multiples of a given number, e.g., which of these are multiples of 4? (0, 4, 6, 8, 14)	• Identify and list the multiples of a given number.		
• Define and give an example of prime and composite numbers.	• Describe the difference between prime and composite numbers.	• Identify prime numbers when given a set of numbers.	• Identify and list prime numbers. NOTE: Two is the <u>smallest and only even</u> prime number. It is important that students		

	LEVELS OF PROGRESSION BELOW THE BENCHMARK Foundational ⟷ Approaching the Benchmark			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			• Identify composite numbers when given a set of numbers.	know that one is **not** a prime number. • Identify and list composite numbers.		
			• Use manipulatives, e.g., base-ten blocks, Digi-Blocks, fraction charts/bars, pattern blocks to identify equivalent fractions.	• Identify the decimal form of commonly used fractions. • Identify equivalent fractions of commonly used fractions. • Identify the fraction form of commonly used decimals.	**MA.4.1.3** **Identify equivalent forms of commonly used fractions and decimals** *NOTE: "Commonly used" fractions are halves, thirds, fourths, and tenths.*	• Convert commonly used decimals into fractions. • Convert commonly used fractions into decimals.
Gr. 3	• Use manipulatives (base-ten blocks, Digi-Blocks, 100s grid, etc.) to represent numbers from hundredths to ten-thousand.	• Identify the digit in a given place value position of a number from the hundredths to the ten-thousands, e.g., given 62,345.08, what digit is in the tenths place? (0) • Identify the value of digits in different place value positions of a number, e.g., given 62,345.08, what is the value of the digit in the hundredths place? (8 hundredths) What is the value of the 2? (2 thousand)	• Read numbers from hundredths to ten-thousands.	• Show different ways to represent place value from hundredths to ten-thousands. • Read a decimal number from hundredth to ten-thousandths, e.g., 1.23 is read as one and twenty-three hundredths. *NOTE: While the benchmark does not require students to be able to "read" a number, it is a foundational requirement subsequent concepts and skills.*	**MA.3.1.1** **Represent place value from hundredths to ten-thousands flexibly**	• Make an equivalent representation of place value for a given representation, e.g., when shown a picture for 30 hundredths, the student can show that this is the same as 3 tenths.

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			• Place a number into one of two categories: odd or even.	• Categorize and explain why a given number is odd or even, e.g., for even numbers: everybody has a partner; the number is divisible by two; the last digit is 0, 2, 4, 6, or 8.	**MA.3.1.2** **Categorize and justify a number as being odd or even** *NOTE: The number must be a whole number.*	
	• Compare pictorial representations of unit fractions. "Unit fraction" means the numerator is one. • Compare unit fractions using the understanding that the smaller the denominator, the larger the value of the fraction.	• Compare two fractions with the same denominator. • Identify a fractional equivalent to a given fraction, e.g., identify 3/4 as equivalent to 9/12.	• Compare two fractions (different denominators) using pictorial representations. • Represent a fractional equivalent to a given fraction, e.g., represent 1/6 pictorially when given the fraction 2/12.	• Compare and order a set of fractions with denominators up to 12 from least to greatest (or greatest to least) using manipulatives, pictorial and/or symbolic representations.	**MA.3.1.3** **Compare and order fractions with denominators up to 12 (e.g., greater than, less than, equal)**	
		• Identify and represent (using manipulatives, pictures, or symbols) the fractions used in a given problem.	• Make a <u>reasonable</u> estimate of the answer to a problem involving fractions, e.g., how many slices of a 12-slice pizza has Shari eaten if she ate ¼ of the pizza? It has to be less than 6 pieces since ¼ is less than ½.	• Solve problems involving fractions with denominators up to 12. *NOTE: This benchmark is about using representations of fractions to solve the problem, and not about performing fraction computations.* Sample problems: • When is one half bigger than another half? (One-half of a large pizza as compared to one-half of a small	**MA.3.1.4** **Use fractions with denominators up to 12 to solve problems**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				pizza.) • How many slices of a 12-slice pizza has Shari eaten if she ate ½ the pizza? • How many slices does a pizza have if Wendy ate two slices, which represents one-third of the pizza?		
Gr. 2			• Show different ways to represent whole numbers up to 500 using concrete materials, pictures, and/or numbers.	• Show different ways to represent whole numbers up to 1000 using concrete materials, pictures and/or numbers by: ○ relating – compare two groups/sets of numbers or groups of objects (more, less, equal) that are equivalent to sums up to 1000. ○ composing – "putting together" or combining numbers	**MA.2.1.1 Represent whole numbers up to 1000 in flexible ways (e.g., relating, composing, and decomposing numbers), including the use of tens and hundreds as units**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				or groups/sets of objects that total sums up to 1000, including using tens and hundreds as units. ○ decomposing – "taking apart" or representing numbers up to 1000 with equivalent groups/sets of smaller whole numbers or groups/ sets of objects, e.g., 345 can be decomposed into 3 groups of hundred, 4 groups of ten, and 5 ones.		
	• Identify the <u>digit</u> in a given place value position of a number, e.g., given 347, what digit is in the tens place? (4)	• Identify the <u>value</u> of each digit in a number, e.g., given 347, identifies the 3 as representing 300 or 3 hundreds, the 4 as 40 or 4 tens, and the 7 as 7 ones.	• Use a place value chart to compare numbers. • Order numbers up to 1000, e.g., order numbers 586, 568, 865, 68, 688 from least to greatest or greatest to least.	• Compare whole numbers up to 1000 and describe the relationship between them, i.e., which number is greater than, less than or equal to when given two or more numbers.	**MA.2.1.2 Compare whole numbers up to 1000 using words (e.g., greater than, less than, equal to)**	
		• Represent fractions with denominators no larger than ten using pictures or models.	• Represent fractions with denominators no larger than ten using pictures, numbers,	• Represent fractions with denominators no larger than ten interchangeably	**MA.2.1.3 Represent fractions with denominators no larger than ten using**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
		• Represent the value of one as a fraction, e.g., five marbles out of a group of five marbles equals 5/5 or 1; OR 8 of the 1/8 pieces of a cake can be combined to make 8/8 or 1.	words, OR models.	between all of the following ways: parts of a set, number symbols, word forms, models, e.g., area diagram such as an array of tiles, number lines.	**pictures, numbers, words, or models**	
Gr. 1	• Count whole numbers up to100 by 10s. OR • Count whole numbers up to 100 by 1s.	• Count whole numbers up to 100 by 10s. AND • Count whole numbers up to 100 by 1s.	• Count whole numbers up to 100 in a variety of ways (by 1s, 5s and 10s).	• Count whole numbers up to 100 in a variety of ways, e.g., skip counts by 2s, 5s, 10s.	**MA.1.1.1 Count whole numbers up to 100 in a variety of ways (e.g., skip counts by 2's, 5's, 10's)**	• Count whole numbers in a variety of ways over 100 and skip count by 25s.
	• Identify a unit fraction as **equal** parts of a whole. *NOTE: A "unit fraction" has a numerator of one.*	• Select the numeric representation and picture/object representation for one-half. • Identify the numerator as the "part" and the denominator as the "whole."	• Represent the value one as a fraction. e.g., *three marbles out of a group of three marbles equals 3/3 or 1; OR 4 of the ¼ pieces of a cake can be combined to make 4/4 or 1.*	• Select the numeric representation and picture/object representation for a given simple fraction e.g., one-half, one-third, one-fourth, two-thirds, three-fourths. *NOTE: For this grade level a "simple fraction" is a fraction whose numerator and denominator are whole numbers.*	**MA.1.1.2 Identify representations of simple fractions (e.g., one-half, one-third, one-fourth)**	• Provide a representation when given a simple fraction.
			• Show different ways to represent whole numbers up to 50 using concrete materials, pictures, and/or numbers.	• Show different ways to represent whole numbers up to 100 using concrete materials, pictures and/or numbers by: ○ relating – compare two groups/sets of numbers or groups of objects (more, less, equal) that are equivalent to sums	**MA.1.1.3 Represent whole numbers up to 100 in flexible ways (e.g., relating, composing, and decomposing numbers), including the use of tens as a unit**	• Show different ways to represent whole numbers over 100 using concrete materials, pictures, and/or numbers.

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				up to 100. ○ composing – "putting together" or combining numbers or groups/sets of objects that total sums up to 100, including using tens as a unit. ○ decomposing – "taking apart" or representing numbers up to 100 with groups/sets of smaller whole numbers or groups/ sets of objects, e.g., 45 can be decomposed into 4 groups of 10, and 5 ones.		
Gr. K	• Rote count up to ten.	• Count with a 1 to 1 correspondence (1 to 1 pointing) up to 15. • Compare sets of objects that are distinctly different in	• Relate a small set of objects to a number. e.g., if student sees this "★★★" he/she says, "three" without counting one, two, three. • Compare sets of objects that are closer in size, e.g., given sets	• Count the number of objects in a group (up to 30 objects). • Compare two or more groups/sets of objects and tell which group	**MA.K.1.1 Count and compare groups of objects up to 30 according to the number of objects in each group**	• Given a set of objects, student can count on from a smaller set, i.e. given a set of eight cubes the student can identify five and count up six, seven, eight.

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
		size, e.g., given sets of two and eight objects, student is able to identify the set of eight as more.	of four and five objects, students can distinguish larger set.	has more/less.		
	• Show different ways to represent whole numbers using concrete materials, pictures and/or numbers up to ten.	• Show different ways to represent whole numbers using concrete materials, pictures, and/or numbers up to 15.	• Show different ways to represent whole numbers using concrete materials, pictures, and/or numbers up to 20.	• Show different ways to represent whole numbers using concrete materials, pictures and/or numbers up to 30 by: ○ relating – compare two groups/sets of numbers or groups of objects (more, less, equal) that are equivalent to sums up to 30. ○ composing – "putting together" or combining numbers or groups/sets of objects that total sums up to 30. ○ decomposing – "taking apart" or representing number up to 30 with groups/sets of smaller whole numbers or groups/sets of objects, e.g., 20 can be decomposed into one group of 10 and two groups of 5 (20 = 10 + 5 + 5).	**MA.K.1.2 Represent whole numbers up to 30 in flexible ways e.g., relating, composing, and decomposing numbers**	• Show different ways to represent whole numbers using concrete materials, pictures, and/or numbers over 30.

Numbers & Operations: K-8

CONTENT STANDARD #2 – OPERATIONS SENSE

Understand the meaning of operations and how they relate to each other

Topic: Operations & Operation Properties

Understanding(s): *Students will understand that…*

- Mathematical operations can be modeled in a variety of ways and describe events/situations.
- Equivalent representations for the same value are used for different purposes.
- Symbolic statements can be manipulated by mathematical rules to produce equivalent statements.
- Equivalent representations for the same value may help to make calculations simpler.
- Understanding number theory concepts helps to represent numbers in different ways.

Essential Question(s):

- How does one ensure common understanding in mathematics?
- How does one apply procedural knowledge to solve problems?

Knowledge: *Students will know…*

- Addition.
- Subtraction.
- Multiplication.
- Division.
- Associative property for addition and multiplication.
- Commutative property for addition and multiplication.
- Distributive property of multiplication over addition.
- Inverse relationships between addition and subtraction.
- Inverse relationships between multiplication and division.
- Inverse relationships between squares and square roots.
- Inverse relationships between cubes and cube roots.
- Percents.
- Decimals.
- Fractions.

Skill(s): *Students will be able to…*

- Order of operations.
- Arithmetic operations using whole numbers.
- Operations using fractions.
- Operations using decimals.
- Operations using percents.
- Operations with integers.

Grade	Reference	Benchmark
Grade 8	MA.8.2.1	Apply the order of operations when calculating with rational numbers
	MA.8.2.2	Demonstrate the inverse relationship between square numbers and square roots, and cubes and cubed roots
Grade 7	MA.7.2.1	Describe situations involving arithmetic operations with integers
	MA.7.2.2	Apply the order of operations when calculating with rational number, excluding exponents
	MA.7.2.3	Apply the inverse relationship between addition and subtraction, and between multiplication and division, to solve one-step equations
Grade 6	MA.6.2.1	Apply the order of operations when calculating with whole numbers
	MA.6.2.2	Use the operation properties to simplify computations with fractions, decimals, and percents
Grade 5	MA.5.2.1	Apply the inverse relationship between addition and subtraction, and multiplication and division, to solve problems

Grade	Reference	Benchmark
	MA.5.2.2	Describe situations involving multiplication and division of fractions and decimals
Grade 4	MA.4.2.1	Describe situations involving addition and subtraction of fractions and decimals
	MA.4.2.2	Use associative, commutative, and distributive properties as they apply to operations involving whole numbers
	MA.4.2.3	Apply the properties of zero and one as they relate to addition, subtraction, multiplication, and division
Grade 3	MA.3.2.1	Recognize situations involving multiplication and division of whole numbers and represent the situation with a number sentence
	MA.3.2.2	Select and apply various meanings and representations of multiplication and division
	MA.3.2.3	Demonstrate that multiplication and division of whole numbers can undo each other
	MA.3.2.4	Use properties of addition of whole numbers (e.g., associative, commutative) to solve problems
Grade 2	MA.2.2.1	Recognize situations involving addition and subtraction and represent the situation with a number sentence
	MA.2.2.2	Demonstrate multiplication as repeated addition of equal groups
	MA.2.2.3	Demonstrate division as "separating equal groups"
Grade 1	MA.1.2.1	Demonstrate that addition and subtraction of whole numbers can undo each other
Grade K	MA.K.2.1	Demonstrate addition as “putting together” or “combining sets”
	MA.K.2.2	Demonstrate subtraction as “taking away,” “separating sets,” or “counting back”

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 8				• Calculate a given numeric expression involving rational numbers by following the order of operation. *NOTE: Exponents are positive integers.*	**MA.8.2.1** **Apply the order of operations when calculating with rational numbers**	• Calculate a given numeric expression involving rational numbers by following the order of operation. *NOTE: Exponents include zero and negative integers.*
	• Expand x^2 as (x)(x) and x^3 as (x)(x)(x), and vice versa.	• Calculate squares and cubes.	• Calculate square roots, squares, cube roots, and cubes of numbers.	• Show that you can "undo" squaring a number by taking the square root of the square, e.g., $\sqrt{3^2} = 3$. • Show that you can "undo" taking the square root of a number by squaring the square root, e.g., $(\sqrt{16})^2 = 16$. • Show that you can "undo" cubing a number by taking the cube root of the cube. • Show that you can "undo" taking the cube root of a number by cubing the cube root.	**MA.8.2.2** **Demonstrate the inverse relationship between square numbers and square roots, and cubes and cubed roots**	
Gr. 7		• Match the situation to its corresponding equation that involves a combination of operations with integers.	• Describe a situation (create a problem) that involves: ○ addition with integers. ○ subtraction with integers. ○ multiplication with integers. ○ division with	• Describe a situation (create a problem) that involves: ○ combination of operations (add, subtract, multiply, divide) with integers.	**MA.7.2.1** **Describe situations involving arithmetic operations with integers**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			integers.			
				• Calculate a given numeric expression involving rational numbers by following the order of operations.	**MA.7.2.2 Apply the order of operations when calculating with rational number, excluding exponents**	
				• Apply inverse relationships to solve one-step equations involving addition, subtraction, multiplication, and division of integers, e.g., to solve the equation x + 8 = -3, the student writes x = -3 – 8, and then x = -11.	**MA.7.2.3 Apply the inverse relationship between addition and subtraction, and between multiplication and division, to solve one-step equations** *NOTE: This is similar to MA.5.2.1 but now includes integers and rational numbers.*	
Gr. 6			• Recite the order of operations. **P** - **P**arenthesis **E** - **E**xponent **MD** - **M**ultiplication/**D**ivision (left to right) **AS** - **A**ddition/**S**ubtraction (left to right)	• Calculate a given numeric expression (with whole numbers only) by following the order of operations.	**MA.6.2.1 Apply the order of operations when calculating with whole numbers**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
		• Apply operation properties with percents.	• Apply operation properties with decimals and percents.	• Apply operation properties (such as the associative, commutative, and distributive properties) to make it easier to perform computations with decimals, fractions, and percents, e.g., ○ 22.5 x 4 can be calculated more efficiently by using the distributive property: 22.5 x 4 = (22 + 0.5) x 4 = (22 x 4) + (0.5 x 4) = 88 + 2 = 90 ○ $1\frac{1}{3}$ x 9 = $(1 + \frac{1}{3})$ x 9 = 1 x 9 + $\frac{1}{3}$ x 9 = 9 + 3 = 12	**MA.6.2.2** **Use the operation properties to simplify computations with fractions, decimals, and percents**	
Gr. 5			• Describe the inverse of addition as subtraction, and vice versa. • Describe the inverse of multiplication as division, and vice versa.	• Apply inverse relationships to solve situations involving one-step equations, e.g., ○ for w – 5 = 10, the student rewrites it as	**MA.5.2.1** **Apply the inverse relationship between addition and subtraction, and multiplication and**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK Foundational ⟷ Approaching the Benchmark			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				w = 10 + 5. ○ for w ÷ 3 = 10, the student rewrites it as w = 10 x 3.	**division, to solve problems**	
		• Match number sentences using fractions or decimals to given situations using multiplication and division.	• Create a number sentence using fractions or decimals when given a situation using multiplication and division.	• Describe a situation (create a problem) involving multiplication with fractions and decimals. • Describe a situation (create a problem) involving division with fractions and decimals.	**MA.5.2.2** **Describe situations involving multiplication and division of fractions and decimals**	
Gr. 4		• Match number sentences using fractions or decimals to given situations using addition and subtraction.	• Create a number sentence using fractions or decimals when given a situation using addition and subtraction.	• Describe a situation (create a problem) involving addition and subtraction of fractions and decimals.	**MA.4.2.1** **Describe situations involving addition and subtraction of fractions and decimals**	
		• Identify and define the distributive property of multiplication over addition (getting the same answer when you multiply a group of numbers by another number as when you do each multiplication separately), e.g., the price for movie tickets is $7.00 per person. If three girls and five boys buy tickets, the total cost could be calculated as: (7 x 3) + (7 x 5) = 21 + 35 = 56	• Identify the property as associative, commutative, or distributive when given a numeric situation. • Recognize that numbers can be multiplied or added in any order but is not possible with subtraction or division (subtraction and division are NEITHER commutative NOR associative).	• Apply associative, commutative, and distributive properties to make it easier to perform computations, e.g., ○ 25 x 3 x 4 can be calculated more efficiently by using the associative and commutative property: 25 x 3 x 4 = 25 x (3 x 4) = 25 x (4 x 3) = (25 x 4) x 3 = 100 x 3 = 300. ○ 28 x 5 might be easier to compute by applying the distributive property:	**MA.4.2.2** **Use associative, commutative, and distributive properties as they apply to operations involving whole numbers**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
		OR 7(3 + 5) = 7 x 8 = 56		28 x 5 = (20 + 8) x 5 = (20 x 5) + (8 x 5) = 100 + 40 = 140.		
				• Use the following properties of zero to make quick mental calculations: ○ zero added to any number is equal to that number. ○ zero subtracted from any number is equal to that number. ○ a number multiplied by zero is equal to zero. ○ zero divided by any non-zero number is equal to zero. ○ DIVISION BY ZERO IS IMPOSSIBLE. • Use the following properties of one to make quick mental calculations: ○ any number multiplied by one results in the original number. ○ any number divided by one results in the original number.	**MA.4.2.3** **Apply the properties of zero and one as they relate to addition, subtraction, multiplication, and division**	
Gr. 3		• Recognize if a problem calls for multiplication or division.	• Select the appropriate number sentence given a variety of choices to match the situation.	• Write an appropriate number sentence (equation) for the given multiplication or division problem situation.	**MA.3.2.1** **Recognize situations involving multiplication and division of whole**	• Create situations involving multiplication and division, and represent with a number sentence.

LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
Foundational ⟷ Approaching the Benchmark					
Students will	**Students will**	**Students will**	**Students will**		**Students will**
				numbers and represent the situation with a number sentence	
		• Show different ways to represent multiplication. e.g., arrays/area models, repeated addition. • Show different ways to represent division, e.g., arrays/area models, repeated subtraction, groupings/partitions, sharing, $\frac{a}{b}$ read as "*a* divided by *b*."	• Read a problem situation and appropriately represent the situation with a multiplication/division model, e.g., Jane gave three pencils to each of her four friends (student might draw four groups of three); Jim arranged the chairs in three rows with four chairs in a row (student might draw a 3-by-4 array of chairs).	**MA.3.2.2 Select and apply various meanings and representations of multiplication and division**	
		• Generate fact families involving multiplication and division. e.g., 3 x 4 = 12 4 x 3 = 12 12 ÷ 4 = 3 12 ÷ 3 = 4	• Show "undoing" multiplication by dividing the product by one of the factors. e.g., 3 x 4 = 12, so 12 ÷ 3 = 4. • Show "undoing" division by multiplying the quotient by the divisor, e.g., 12 ÷ 3 = 4, so 4 x 3 = 12.	**MA.3.2.3 Demonstrate that multiplication and division of whole numbers can undo each other**	
• Identify and define the commutative property as situations where the order of addends can be switched but the sum remains the same,	• Identify and define the associative property as being able to change the grouping of addends but the sum remains the same,	• Recognize that numbers can be added in any order, but is not possible with subtraction, i.e., subtraction is NEITHER	• Apply commutative and/or associative properties to solve a problem that requires addition.	**MA.3.2.4 Use properties of addition of whole numbers (e.g., associative, commutative)**	• Mentally apply commutative and/or associative properties to solve a problem that requires addition. e.g., when solving a problem

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
	e.g., 2 + 3 = 3 + 2.	e.g., (3 + 4) + 6 = 3 + (4 + 6). *NOTE: This is the first introduction to order of operations where the operations in the parentheses need to be done first.*	commutative NOR associative). • Demonstrate the associative and commutative properties by rewriting a numerical expression.		**to solve problems**	that involves adding 13 + 24 + 7, the student mentally applies the commutative and associative properties to add 13 + 7 first, then add 24.
Gr. 2		• Select the appropriate equation given a situation.	• Recognize when a situation calls for addition. • Write an appropriate equation (number sentence) for the given addition problem.	• Recognize when a situation calls for addition or subtraction. • Write an appropriate equation (number sentence) for the given addition or subtraction problem situation.	**MA.2.2.1 Recognize situations involving addition and subtraction and represent the situation with a number sentence**	• Create a situation using addition and subtraction, and provide the corresponding number sentence.
			• Demonstrate how to add a number repeatedly, e.g., 4 + 4 = 8, + 4 more = 12, + 4 more = 16.	• Demonstrate multiplication as repeated addition of equal groups, e.g., 4 x 3 = 4 + 4 + 4 = 12.	**MA.2.2.2 Demonstrate multiplication as repeated addition of equal groups**	
			• Demonstrate division as separating equal groups, with numbers ten and below through storytelling and the use of manipulatives. *NOTE: Build understanding of division w/ smaller numbers first.*	• Demonstrate division as "separating equal groups, e.g., start with 15 objects and share them equally among 3 children. Each child would receive 5 objects each.	**MA.2.2.3 Demonstrate division as "separating equal groups"**	• Demonstrate division as "separating equal groups with a remainder, e.g., start with 15 objects and share them equally among 4 children. Each child would receive 3 objects each with 3 leftover.
Gr. 1			• Provide examples of using opposites (e.g., up/down, under/over, off/on, in/out) to return situations to their original status.	• Show how subtracting, e.g., "taking away," the same number that was added to a given number results in the original number,	**MA.1.2.1 Demonstrate that addition and subtraction of whole numbers can undo**	• Represent "undoing" adding/subtracting a number, numerically.

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			NOTE: This is to introduce inverse relationships in mathematics.	e.g., 10 + 2 – 2 = 10. • Show how adding the same number that was subtracted from a given number results in the original number e.g., 10 – 2 + 2 = 10.	**each other** *NOTE: Use of fact families can be helpful to the understanding of inverse relationships, e.g.,* *5 + 2 = 7* *2 + 5 = 7* *7 – 2 = 5* *7 – 5 = 2*	
Gr. K			• Put together or combine groups/sets of objects or pictures that result in a total amount or sum.	• Put together or combine two groups/sets of objects, pictures, or **numbers** that result in a total amount or sum.	**MA.K.2.1 Demonstrate addition as "putting together" or "combining sets"**	• Put together or combine three or more groups/sets of objects, pictures, or **numbers** that result in a total amount or sum.
		• Count backwards from ten.	• Use groups/sets of objects or pictures to show how many are left by "taking away," "separating sets," that result in a difference. • Count backwards from 15.	• Use groups/sets of objects, pictures, or numbers, to show how many are left by "taking away," "separating sets," (these are synonymous terms) **or** "counting backwards" from the original set, e.g., starting from 6, counting backwards "5, 4" would be represented by 6 – 2 = 4.	**MA.K2.2 Demonstrate subtraction as "taking away," "separating sets," or "counting back"**	• Use **numbers only** to represent subtraction.

Numbers & Operations: K-8

CONTENT STANDARD #3 – COMPUTATION STRATEGIES

Use computational tools and strategies fluently and, when appropriate, use estimation

Topic: Computational Fluency & Estimation

Understanding(s): *Students will understand that…*

- Mathematical operations can be modeled in a variety of ways, and describe events/situations.
- Equivalent representations for the same value are used for different purposes.
- An estimate will sometimes suffice rather than an exact answer.
- Symbolic statements can be manipulated by mathematical rules to produce equivalent statements.
- Equivalent representations for the same value may help to make calculations simpler.

Essential Question(s):

- How does one ensure computational fluency (i.e. efficiency, flexibility and accuracy)?
- How does one ensure solutions are mathematically reasonable?

Knowledge: *Students will know…*

- Estimation.
- Fractions.
- Decimals.
- Exponents.
- Integers.

Skill(s): *Students will be able to…*

- Recall addition and subtraction facts up to 20.
- Recall multiplication facts up to 12 X 12.
- Use strategies to add, subtract, multiply and divide whole numbers, fractions, decimals, numbers with whole number exponents, and integers.
- Estimate prior to computing.

Grade	Reference	Benchmark
Grade 8	MA.8.3.1	Add, subtract, multiply, and divide numbers with whole number exponents
	MA.8.3.2	Estimate a reasonable range (i.e., upper and lower limit) for the solution to a problem
Grade 7	MA.7.3.1	Add, subtract, multiply, and divide integers
	MA.7.3.2	Determine the reasonableness of a solution by comparing the answer to an estimate
Grade 6	MA.6.3.1	Use estimation prior to computing with fractions and decimals and compare the estimation to the actual result
	MA.6.3.2	Recognize situations in which it is more appropriate to estimate than to compute an exact answer
Grade 5	MA.5.3.1	Multiply decimals up to 3 places and divide decimals by whole numbers
	MA.5.3.2	Use a variety of strategies to multiply and divide fractions
Grade 4	MA.4.3.1	Recall all multiplication facts and the corresponding division facts up to 12 x 12
	MA.4.3.2	Select and use appropriate strategies and/or tools (e.g., mental math, calculators, paper/pencil, standard algorithms) for computing whole numbers
	MA.4.3.3	Use a variety of strategies to add and subtract fractions with like and unlike denominators
	MA.4.3.4	Add and subtract decimals to 3 places
	MA.4.3.5	Determine the reasonableness of numerical solutions
Grade 3	MA.3.3,1	Recall multiplication facts from 0 x 0 to 10 x 10
	MA.3.3.2	Use a variety of strategies to solve problems involving addition and subtraction of two- and three-digit numbers
	MA.3.3.3	Estimate the results of whole-number computations
Grade 2	MA.2.3.1	Recall addition facts and their corresponding subtraction facts up to twenty

Grade	Reference	Benchmark
	MA.2.3.2	Use a variety of strategies to solve problems involving addition and subtraction of two-digit numbers
	MA.2.3.3	Estimate the solution of addition and subtraction problems
Grade 1	MA.1.3.1	Recall single-digit addition facts
	MA.1.3.2	Use a variety of strategies to solve number problems involving addition and subtraction (e.g., comparing sets, counting on, counting backwards, doubles, doubles plus one)
Grade K	MA.K.3.1	Use a variety of strategies (e.g., objects, fingers) to add and subtract single-digit whole numbers

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 8	• Simplify terms with exponents. e.g., $3^2 = 9$; $2^3 = 8$.	• Add/subtract terms with exponents by following the order of operations.	• Add/subtract terms with exponents by following the order of operations. • Multiply/divide terms that have the same base, e.g., $2^3 \cdot 2^4 = 8 \cdot 16 = 128$; $2^4 / 2^3 = 16 / 8 = 2$.	• Add/subtract terms with exponents by following the order of operations. e.g., $2^2 + 5^2 = 4 + 25 = 29$. • Multiply terms that have the same base by: ○ Applying the order of operations, e.g., $2^2 \cdot 2^3 = 4 \cdot 8 = 32$ ○ Applying the law of exponents first, e.g., $2^2 \cdot 2^3 = 2^{2+3} = 2^5 = 32$. ○ Expanding the terms before regrouping to make it simpler to multiply, e.g., $2^2 \cdot 2^3 = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) = (2 \cdot 2 \cdot 2 \cdot 2) \cdot 2 = 16 \cdot 2 = 32$. • Divide terms that have the same base by: ○ Applying the order of operations, e.g., $2^5 / 2^3 = 32 / 8 = 4$. ○ Applying the law of exponents first, e.g., $2^5 / 2^3 = 2^{5-3} = 2^2 = 4$. • Expand the terms before regrouping to make it simpler to divide, e.g., $25 / 23 = (2 \cdot 2 \cdot 2 \cdot 2 \cdot 2) / (2 \cdot 2 \cdot 2) = (2 \cdot 2) \cdot (2 \cdot 2 \cdot 2) / (2 \cdot 2 \cdot 2) = 2 \cdot 2 = 4$.	**MA.8.3.1** **Add, subtract, multiply, and divide numbers with whole number exponents**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			• Determine an estimate for the solution of a problem but has difficulty finding a range in which the actual answer falls.	• Determine an estimate for the solution of a problem, and state with certainty a range in which the actual answer falls, e.g., when calculating the length of the hypotenuse, $\sqrt{10}$, the students says that the answer is about 3.1 because $\sqrt{9}$ is between (= 3) and $\sqrt{16}$ (= 4).	**MA.8.3.2 Estimate a reasonable range (i.e., upper and lower limit) for the solution to a problem**	
Gr. 7	.	• Multiply and divide integers.	• Add integers.	• Add integers, subtract integers, multiply integers, and divide integers.	**MA.7.3.1 Add, subtract, multiply, and divide integers**	
				• Estimate the solution to a problem prior to carrying out an actual computation. • Carry out the actual computation, and then use the estimated answer to validate/invalidate computed solution.	**MA.7.3.2 Determine the reasonableness of a solution by comparing the answer to an estimate**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 6			• Replace fractions or decimals with "friendly" numbers in order to mentally compute an estimated answer.	• Replace fractions and decimals with "friendly" numbers in order to mentally compute an estimated answer. • Carry out the actual computation and then use the estimated answer to validate/invalidate computed solution.	**MA.6.3.1 Use estimation prior to computing with fractions and decimals and compare the estimation to the actual result**	
				• Indicate whether a given problem situation requires an exact answer or whether an estimation will suffice.	**MA.6.3.2 Recognize situations in which it is more appropriate to estimate than to compute an exact answer**	
Gr. 5			• Place the decimal point in the right position of a multiplication or division problem involving decimals.	• Multiply decimals up to the thousandths place. • Divide decimals up to the thousandths place by whole numbers.	**MA.5.3.1 Multiply decimals up to 3 places and divide decimals by whole numbers**	• Check the answer to their multiplication/ division problem involving decimals up to the thousandths place (or more) by using estimation.
			• Use an appropriate strategy to multiply and divide fractions.	• Use more than one strategy to multiply and divide fractions, e.g., traditional algorithm, area model, drawing pictures.	**MA.5.3.2 Use a variety of strategies to multiply and divide fractions**	• Explain why a particular strategy was appropriate and why another was not appropriate for that situation.
Gr.				• Recall multiplication	**MA.4.3.1**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
4				facts up to 12 x 12 and their corresponding division facts with automaticity.	**Recall all multiplication facts and the corresponding division facts up to 12 x 12**	
			• Recall what problem-solving tools are available.	• Choose and use a computation strategy/tool that is most appropriate for performing the computation.	**MA.4.3.2 Select and use appropriate strategies and/or tools (e.g., mental math, calculators, paper/pencil, standard algorithms) for computing whole numbers**	• Explain why a particular strategy was appropriate and why another was not appropriate for that situation.
			• Add and subtract fractions with like denominators.	• Add and subtract fractions with like and unlike denominators using more than one strategy, e.g., using drawings/pictures; using fractions strips or other manipulatives; applying the commutative and associative properties. Commutative sample: $\frac{1}{3} + (\frac{2}{3} + \frac{3}{4})$ $= (\frac{1}{3} + \frac{2}{3}) + \frac{3}{4}$ $= 1 + \frac{3}{4}$	**MA.4.3.3 Use a variety of strategies to add and subtract fractions with like and unlike denominators**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				$= 1\frac{3}{4}$		
				Another way to add fractions using common denominators: $\frac{1}{3} + \frac{2}{3} + \frac{3}{4}$ $= \frac{4}{12} + \frac{8}{12} + \frac{9}{12}$ $= \frac{21}{12}$ $= 1\frac{9}{12}$ $= 1\frac{3}{4}$		
			• Add and subtract decimals up to two places. • Estimate numbers to the tenths or hundredths place.	• Add and subtract decimals up to three places.	**MA.4.3.4** **Add and subtract decimals to 3 places**	
				• Determine the reasonableness of numerical solutions based on the context and level of accuracy demanded by the	**MA.4.3.5** **Determine the reasonableness of numerical solutions** *NOTE: MA.4.3.5 should contain solutions involving*	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
				problem or situation, e.g., 25 students are going on a field trip in private vehicles. Each vehicle can take 4 students. How many vehicles are needed? The answer would be 7 vehicles, not 6 vehicles or 6¼ vehicles.	*decimals and fractions.*	
Gr. 3				• Recall multiplication facts with automaticity.	**MA.3.3.1 Recall multiplication facts from 0 x 0 to 10 x 10**	
			• Solve problems involving addition and subtraction of two- and three-digit numbers using one or two strategies.	• Solve problems involving addition and subtraction of two- and three-digit numbers using different strategies, e.g., decomposing numbers into "friendlier" numbers, modeling with manipulatives, and applying the standard algorithm.	**MA.3.3.2 Use a variety of strategies to solve problems involving addition and subtraction of two- and three-digit numbers**	
			• Estimate by truncating before computing, e.g., 304 + 278 is estimated as 300 + 200 = 500. *NOTE: Truncating may not be as accurate as rounding.*	• Round numbers to a particular place value, then compute the estimate, e.g., 304 + 278 is rounded to 300 + 300 = 600. • Check for reasonableness of the actual answer to the estimate.	**MA.3.3.3 Estimate the results of whole-number computations**	

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
Gr. 2		• Recall with automaticity the single-digit addition facts and their corresponding subtraction facts up to 10, e.g., 5 + 5 = 10, 10 – 5 = 5.	• Recall with automaticity the addition facts and their corresponding subtraction facts up to 12, e.g., 8 + 4 = 12, 12 – 8 = 4.	• Understand the single-digit addition facts and their corresponding subtraction facts up to 20 well enough to recall the facts with automaticity.	**MA.2.3.1 Recall addition facts and their corresponding subtraction facts up to twenty**	• Recall addition facts and their corresponding subtraction facts for 21 and up.
			• Solve problems involving addition and subtraction of two-digit numbers using one or two strategies.	• Solve problems involving addition and subtraction of two-digit numbers using several different strategies, e.g., manipulatives, base-ten blocks, drawings, mental math, applying the standard algorithm.	**MA.2.3.2 Use a variety of strategies to solve problems involving addition and subtraction of two-digit numbers**	
			• Use an estimation strategy to determine an answer that should be reasonably close to the actual answer when solving addition problems.	• Use an "estimation strategy," e.g., making tens, rounding using place value concept, comparison to known values, etc., to determine an answer that should be reasonably close to the actual answer when solving addition AND subtraction problems.	**MA.2.3.3 Estimate the solution of addition and subtraction problems**	
Gr. 1		• Recall with automaticity results of adding zero or one to a single-digit number.	• Recall with automaticity results of adding single-digit numbers whose sums are up to ten.	• Recall with automaticity results of adding single-digit numbers.	**MA.1.3.1 Recall single-digit addition facts**	• Recall more complex addition facts, e.g., 16 + 2 = 18.

	LEVELS OF PROGRESSION BELOW THE BENCHMARK			AT THE BENCHMARK	BENCHMARK	ADVANCED
	Foundational ⟷ Approaching the Benchmark					
	Students will	**Students will**	**Students will**	**Students will**		**Students will**
			• Solve addition and subtraction number problems using one strategy. • Solve addition and subtraction number problems where the missing number is the sum or difference, e.g., 3 + 5 = ___ or 8 – 2 = ___.	• Solve addition and subtraction number problems using different strategies. • Solve addition and subtraction number problems where the missing number is not the sum or difference, e.g., 3 + ___ = 8 or ___ – 2 = 6.	**MA.1.3.2** **Use a variety of strategies to solve number problems involving addition and subtraction (e.g., comparing sets, counting on, counting backwards, doubles, doubles plus one)**	• Solve addition and subtraction number problems with more than two terms, e.g., 6 + 1 + ___ = 10 or 4 + 2 – ___ = 1.
Gr. K		• Show how to add single-digit numbers in several different ways.	• Show how to add OR subtract single-digit numbers in several different ways.	• Show how to add AND subtract single-digit numbers in several different ways, e.g., objects, fingers, drawing, or using tally marks.	**MA.K.3.1** **Use a variety of strategies (e.g., objects, fingers) to add and subtract single-digit whole numbers**	• Show how to add AND subtract three or more single digit whole numbers in several ways.